Physics & diagnostics in Tokamak plasmas
-
Thomson scattering

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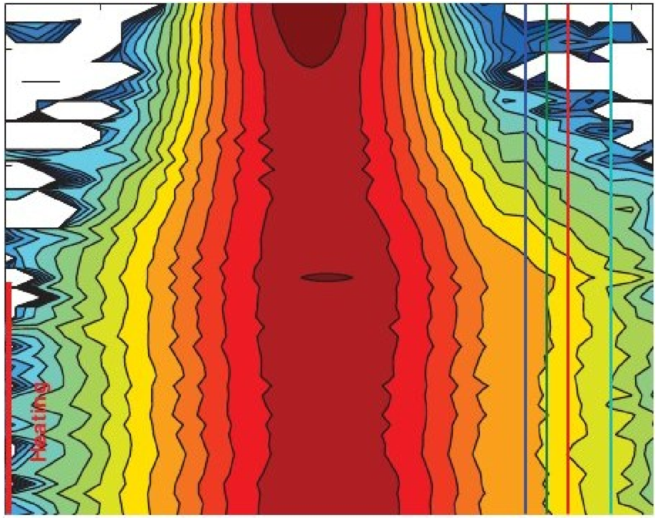


Physics and Diagnostics in Tokamak plasmas

How to measure

- electron and ion
- density, velocity and temperature
- in the plasma core ?

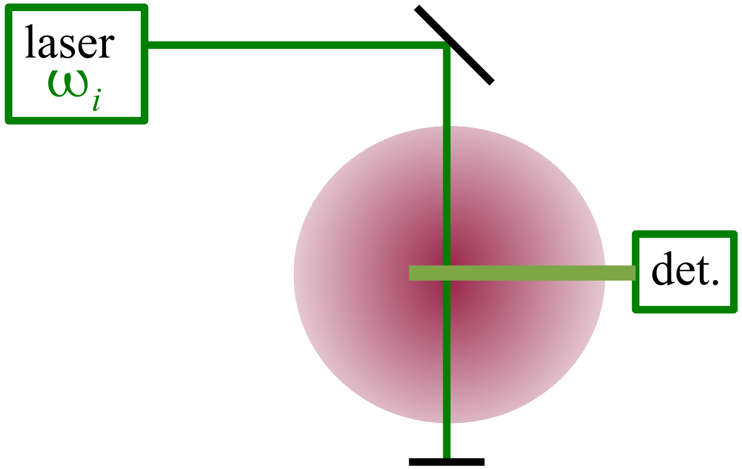
Lecture

- I. Thomson Scattering
  - Thomson Scattering principle
  - Incoherent Thomson Scattering
    - ITS and electron dynamics
    - Relativistic particle case
    - Magnetic field effect
    - ITS applications (T3, JET, ITER)
  - Coherent Thomson Scattering
    - CTS and ion and electron dynamics
    - CTS applications (TEXTOR, ASDEX-U, ITER)

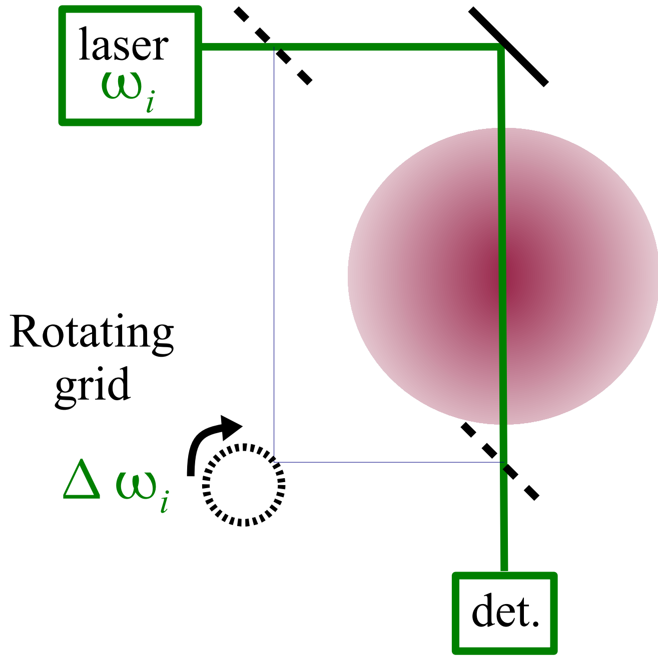
Thomson Scattering principle

Thomson Scattering principle

Scattering is different from interferometry :
instead of detecting the phase shift of the initial beam through the plasma, we look at the light remitted in every other direction by the the plasma.



Thomson Scattering



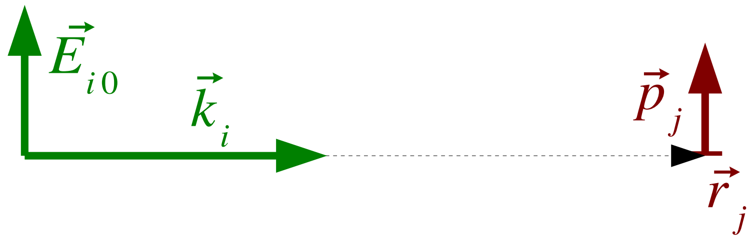
Interferometry

Emitted light intensity will give information on the electron density.
Doppler effect will give information on particle velocity distributions.

Thomson Scattering : free electron scatterer

Simple model : **free electrons** are the scatterers

the scatterers are **non relativistic**, **no magnetic field**



Mono-chromatic transverse linear mono-mode wave : $\vec{k}_i, \lambda_i, \omega_i$ $\omega_i \gg \omega_p$

$$\vec{E}_i(\vec{r},t) = e^{i(\vec{k}_i.\vec{r}-\omega_i t)} \vec{E}_{i0}$$

The incident wave electric field accelerates mainly plasma **free electrons**

$\vec{r}_j(t)$: single electron position

$$m_e \vec{a}_j(t) = -q_e \vec{E}_i(\vec{r}_j(t),t) \qquad q_e > 0$$

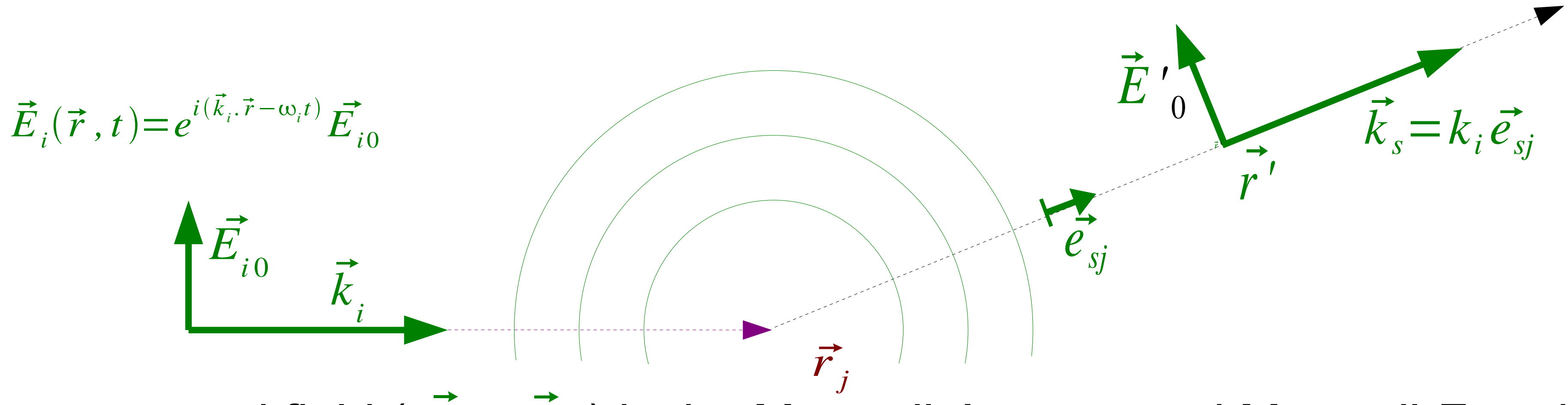
The electron position oscillates at the incident wave frequency :

$$\vec{r}_j(t) = \frac{q_e}{m_e \omega_i^2} \vec{E}_i(\vec{r}_j(t),t)$$

Each single electron oscillation creates a local oscillating dipole :

$$\vec{p}_j(t) = -q_e \vec{d}[\vec{r}_j(t),t] = \frac{-q_e^2}{m_e \omega_i^2} \vec{E}_i(\vec{r}_j(t),t)$$

Thomson Scattering : electric field scattered by a dipole



The scattered field ($\vec{E}_{sj}$, $\vec{H}_{sj}$) is the Maxwell-Ampere and Maxwell-Faraday equation solutions with a dipole as the source :

$$\vec{\nabla}\wedge\vec{H}_{sj}(\vec{r}\,',t)=\partial_t\left(\varepsilon_0\vec{E}_{sj}(\vec{r}\,',t)+\vec{p}_j(t)\,\delta(\vec{r}\,'-\vec{r}_j)\right)\qquad \partial_t=\frac{\partial}{\partial t}$$

$$\vec{\nabla}\wedge\vec{E}_{sj}(\vec{r}\,',t)=-\mu_0\,\partial_t\vec{H}_{sj}(\vec{r}\,',t)$$

Both equation combined in a wave equation with a punctual source :

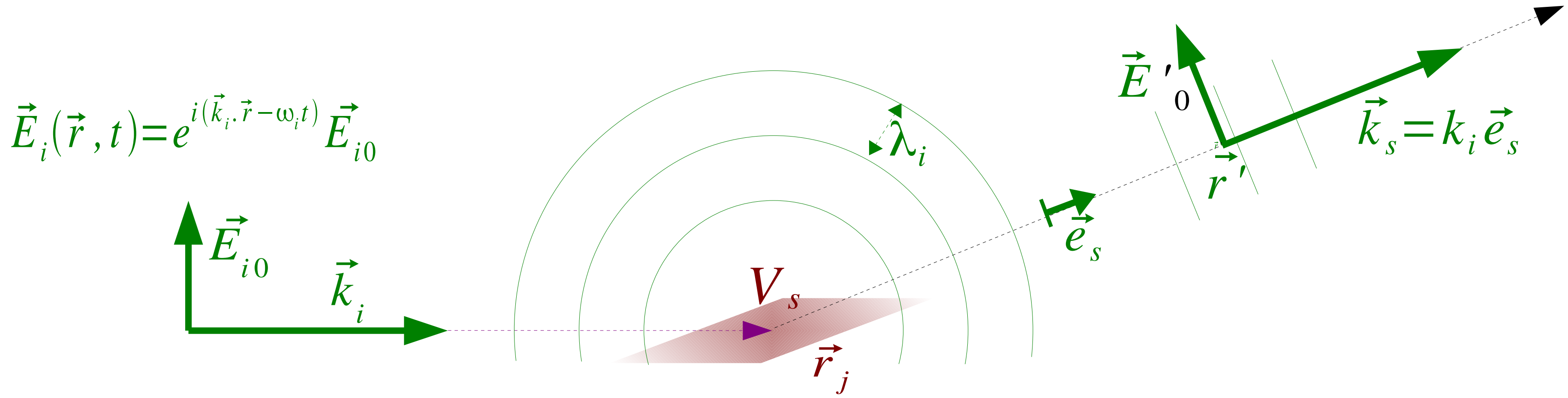
$$-\vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{E}_{sj}(\vec{r}\,',t))+\frac{\omega_i^2}{C^2}\vec{E}_{sj}(\vec{r}\,',t)=-\mu_0\,\vec{p}_j(t)\,\delta(\vec{r}\,'-\vec{r}_j)$$

The wave solution is a Green function corresponding to a spherical wave :

$$\vec{E}_{sj}(\vec{r}\,',t)=\frac{\mu_0 q_e^2}{4\pi m_e}\frac{\mathrm{e}^{ik_i|\vec{r}\,'-\vec{r}_j|}}{|\vec{r}\,'-\vec{r}_j|}\mathrm{e}^{i(\vec{k}_i.\vec{r}_j-\omega_i t)}\vec{e}_{sj}\wedge(\vec{e}_{sj}\wedge\vec{E}_{i0})$$

Thomson scattering is elastic (no energy absorption nor emission) : $|\vec{k}_s|=|\vec{k}_i|$

Thomson Scattering : far field scattered electric field



Single free electron scattered field :

$$\vec{E}_{sj}(\vec{r}',t)=\frac{\mu_0 q_e^2}{4\pi m_e}\frac{e^{ik_i|\vec{r}'-\vec{r}_j|}}{|\vec{r}'-\vec{r}_j|}e^{i(\vec{k}_i.\vec{r}_j-\omega_i t)}\vec{e}_{sj}\wedge(\vec{e}_{sj}\wedge\vec{E}_{i0})$$

Far field approximation :

$|\vec{r}'|\gg|\vec{r}_j|$: observation distance is much larger than scattering volume V_s dimension.

V_s is defined by crossing of **incident beam** and detector **antenna beam**.

$|\vec{r}'|\gg\lambda_i$: observation distance is much larger than the incident wave wavelength.

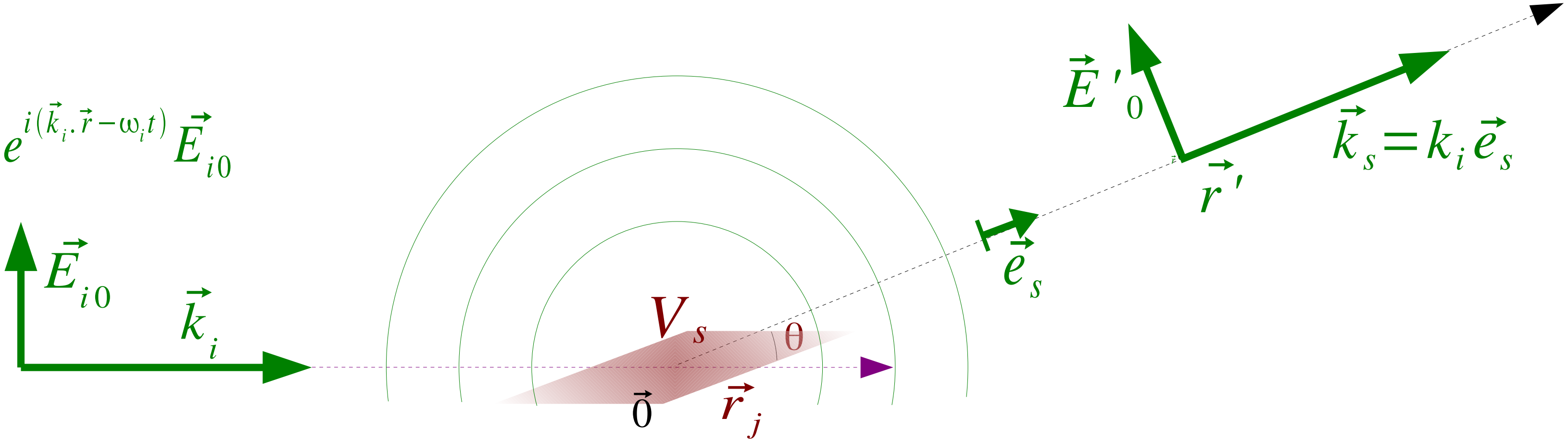
→ One common observation direction : $\vec{e}_{sj}\sim\vec{e}_s$ $\quad\frac{1}{|\vec{r}'-\vec{r}_j|}\sim\frac{1}{r'}$

→ Spherical locally considered as plane wave : $k_i|\vec{r}'-\vec{r}_j|\sim k_i\vec{e}_s.(\vec{r}'-\vec{r}_j)$

$$\vec{E}_{sj}(\vec{r}',t)=\frac{\mu_0 q_e^2}{4\pi m_e}\frac{e^{i\vec{k}_s.\vec{r}'}}{r'}e^{-i\omega_i t}\vec{e}_s\wedge(\vec{e}_s\wedge\vec{E}_{i0})\, e^{-i(\vec{k}_s-\vec{k}_i).\vec{r}_j(t)}$$

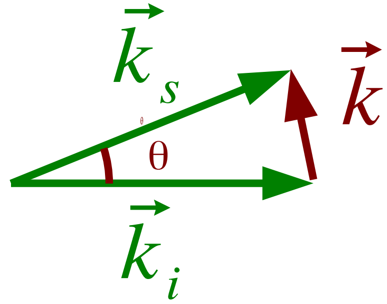
Thomson Scattering : scattered electric field factors

$$\vec{E}_i(\vec{r},t)=e^{i(\vec{k}_i.\vec{r}-\omega_i t)}\vec{E}_{i0}$$



Single free electron scattered field with far field approximation :

$$\vec{E}_{sj}(\vec{r}',t)=\frac{\mu_0 q_e^2}{4\pi m_e}\frac{e^{i\vec{k}_s.\vec{r}'}}{r'}e^{-i\omega_i t}\,\vec{e}_s\wedge(\vec{e}_s\wedge\vec{E}_{i0})\,e^{-i(\vec{k}_s-\vec{k}_i).\vec{r}_j(t)}$$



$$\vec{k}=\vec{k}_s-\vec{k}_i$$

Scattering wave vector (Bragg's condition)

$$k=2k_i\sin(\theta/2)$$

$e^{-i(\vec{k}_s-\vec{k}_i).\vec{r}_j(t)}=e^{-i\vec{k}.\vec{r}_j(t)}$: **the electron specific phase** function of **the scattering wave vector**

$\vec{E}'_0=\vec{e}_s\wedge(\vec{e}_s\wedge\vec{E}_{i0})$: wave polarization modification

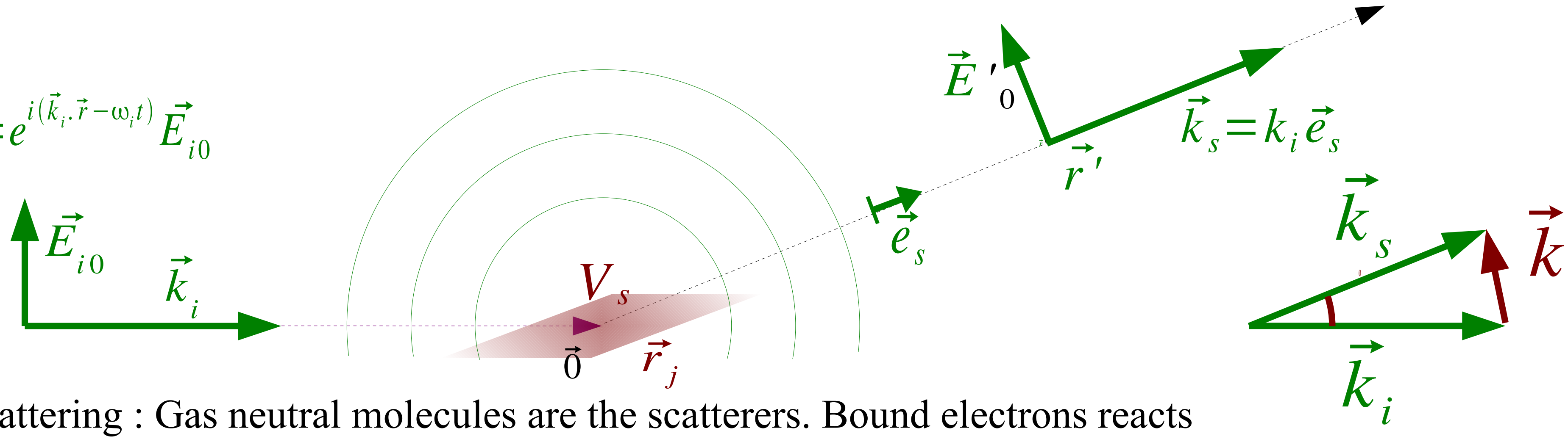
$e^{-i\omega_i t}$: source frequency phase

$\dfrac{e^{i\vec{k}_s.\vec{r}'}}{r'}$: wave spherical structure

$\dfrac{\mu_0 q_e^2}{4\pi m_e}=r_0=2{,}8\ 10^{-15}\,m$: electron classical radius

Thomson Scattering vs Rayleigh Scattering

$\vec{E}_i(\vec{r},t)=e^{i(\vec{k}_i.\vec{r}-\omega_i t)}\vec{E}_{i0}$



Rayleigh scattering : Gas neutral molecules are the scatterers. Bound electrons reacts to the initial wave. **Neutral molecules get polarized** :

$$\vec{p}_j(t)=\alpha_j\varepsilon_0\vec{E}_i(\vec{r}_j(t),t)$$

α_j : molecule polarizability : $\alpha_{air}=4\pi\,1.65\;10^{-30}m^3$

Far field approximation of the Rayleigh scattered electric field :

$$\vec{E}_{sj}(\vec{r}\,',t)=\frac{\pi}{n_W^2\lambda_i^2}\frac{\mathrm{e}^{i\vec{k}_s.\vec{r}\,'}}{r'}\mathrm{e}^{-i\omega_i t}\vec{e}_s\wedge(\vec{e}_s\wedge\vec{E}_{i0})\;\alpha_j\mathrm{e}^{-i\vec{k}.\vec{r}_j}$$

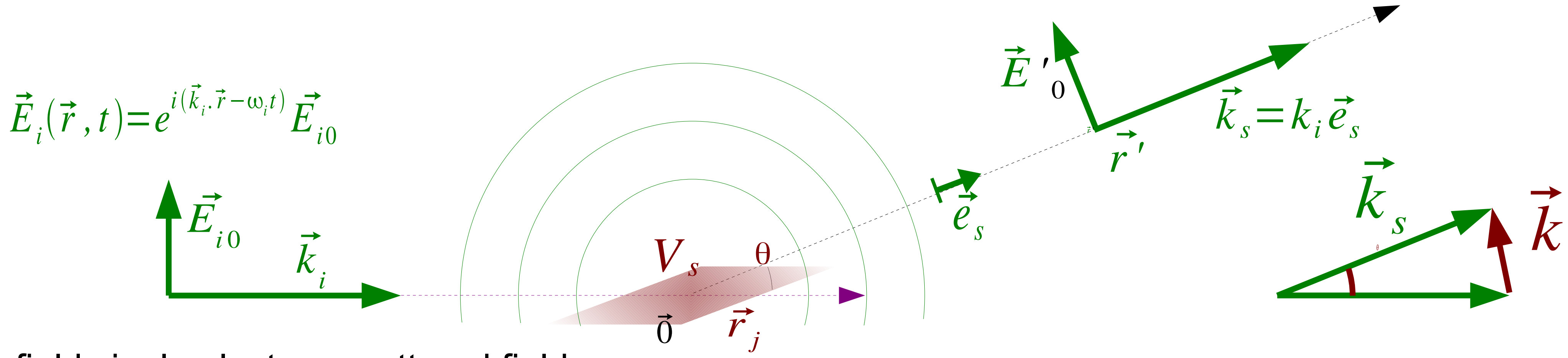
r_0 the electron classical radius is replaced with the averaged Rayleigh radius : $r_R=\frac{\pi\langle\alpha\rangle}{n_W^2\lambda_i^2}$

n_W : medium refraction index

$r_{Rair}\sim r_0$ $\qquad$ $\lambda_i\sim 0.15\,\mu m$

The Rayleigh radius depends on initial wave wavelength : it is larger for smaller wavelength.
The Rayleigh radius is smaller than r_0 for wavelength larger than UV wavelengths.
Rayleigh scattering might be used to **calibrate** Thomson scattering diagnostic.

Thomson scattering anisotropy



Far field single electron scattered field :

$$\vec{E}_{sj}(\vec{r}',t)=r_0\frac{e^{i\vec{k}_s.\vec{r}'}}{r'}e^{-i\omega_i t}\,\vec{e}_s\wedge(\vec{e}_s\wedge\vec{E}_{i0})\,e^{-i\vec{k}.\vec{r}_j(t)}$$

The emission is not isotropic. It depends on the angle between the wave polarization and the observation direction :

$$\theta_{pol}=\pi/2-\widehat{(\vec{e}_s,\vec{E}_{i0})}$$

$$E'_{i0}=\left|\vec{e}_s\wedge(\vec{e}_s\wedge\vec{E}_{i0})\right|=E_{i0}\cos\theta_{pol}$$

When the wave polarization is in the scattering plane, this angle is the scattering angle :

$$\theta_{pol}=\theta \qquad E'_{i0}=E_{i0}\cos\theta$$

When the wave polarization is perpendicular to the scattering plane, this angle is null :

$$\theta_{pol}=0 \qquad E'_{i0}=E_{i0}$$

The scattered electric field is reduced when this angle goes to 90°.

Thomson scattering cross section per electron

Far field single electron scattered field :

$$\vec{E}_{sj}(\vec{r}\,',t)=r_0\frac{e^{i\vec{k}_s.\vec{r}\,'}}{r'}\,e^{-i\omega_i t}\,\vec{e}_s\wedge(\vec{e}_s\wedge\vec{E}_{i0})\,e^{-i\vec{k}.\vec{r}_j(t)}$$

The scattered electric field amplitude :

$$|\vec{E}_{sj}(\vec{r}\,',t)|=r_0\frac{1}{r'}E_{i0}\cos\theta_{pol}$$

For each **solid angle** scattered direction (θ,φ), the electromagnetic **scattered intensity** compared to the initial wave intensity :

$$\frac{d\sigma_j}{d\Omega_s}(\theta,\varphi)=\frac{1}{I_i}\frac{dI_{sj}(\theta,\varphi)}{d\Omega_s}=\frac{r'^2 I_{sj}}{I_i}=\frac{r'^2\langle|E_{sj}|^2\rangle}{\langle|E_{i0}|^2\rangle}$$

So :

$$\frac{d\sigma_j}{d\Omega_s}(\theta,\varphi)=r_0^2\cos^2\theta_{pol}$$

Averaged for both possible initial polarizations :

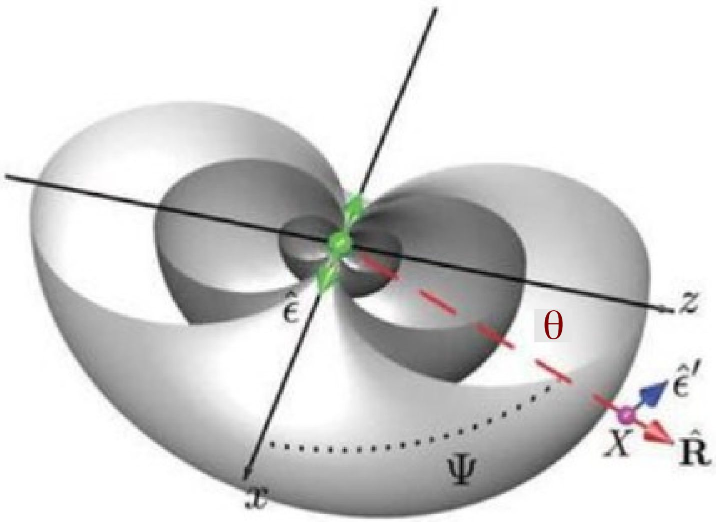
$$\frac{d\sigma_j}{d\Omega_s}(\theta,\varphi)=r_0^2\frac{1+\cos^2\theta}{2}$$

Integrated for all solid angles :

$$\sigma_{Tj}=\int_{-\pi/2}^{\pi/2}d\theta\int_0^{2\pi}\frac{d\sigma_j}{d\Omega_s}(\theta,\varphi)\sin\theta\,d\varphi$$

The Thomson scattering **cross section** for a single electron :

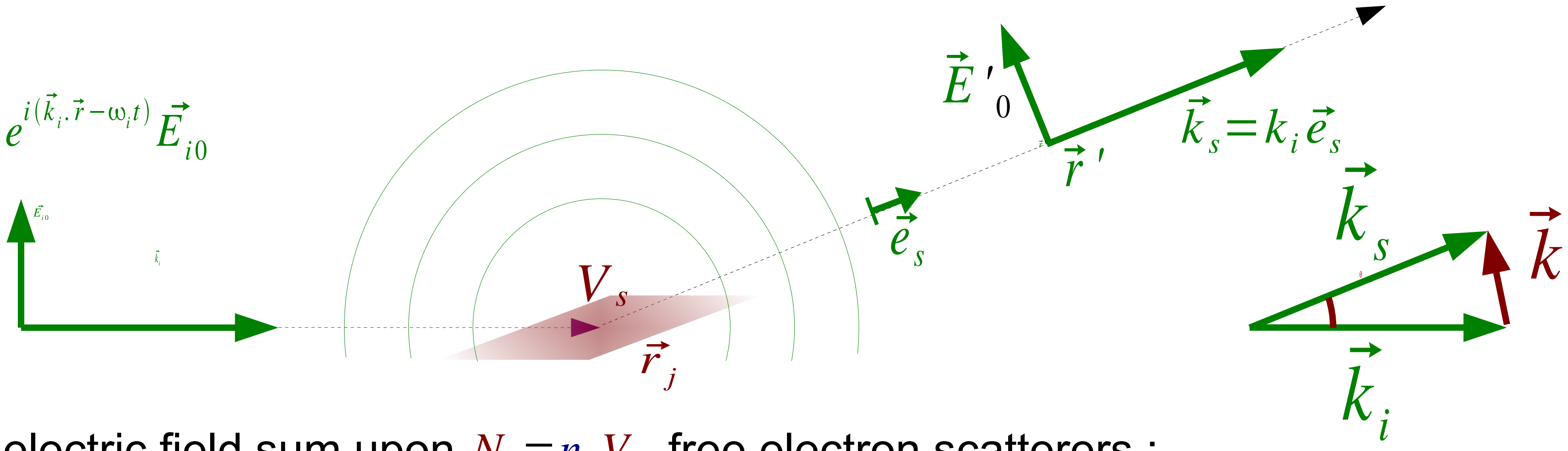
$$\sigma_{Tj}=\frac{8\pi}{3}r_0^2$$



Incoherent Thomson Scattering

Thomson Scattering : scatterer sum

$$\vec{E}_i(\vec{r},t)=e^{i(\vec{k}_i.\vec{r}-\omega_i t)}\vec{E}_{i0}$$



Scattered electric field sum upon $N_s=n_e V_s$ free electron scatterers :

$$\vec{E}_s(\vec{r}',t)=\sum_{j=1}^{N_s}\vec{E}_{sj}(\vec{r}',t)$$

$$\vec{E}_s(\vec{r}',t)=r_0\frac{\mathrm{e}^{i\vec{k}_s.\vec{r}'}}{r'}\vec{e}_s\wedge(\vec{e}_s\wedge\vec{E}_{i0})\mathrm{e}^{-i\omega_i t}\sum_{j=1}^{N_s}\mathrm{e}^{-i\vec{k}.\vec{r}_j(t)}$$

Each scatterer scattered electric field differs by **the scattering phase**.
The **scattering signal** will be defined by the sum of the scattering phases **:**

$$n_{\vec{k}}(t)=\sum_{j=1}^{N_s}\mathrm{e}^{-i\vec{k}.\vec{r}_j(t)}$$

Incoherent Thomson Scattering : signal correlation

$$n_{\vec{k}}(t)=\sum_{j=1}^{N_s} e^{-i\vec{k}.\vec{r}_j(t)}$$

Scattering signal time correlation :

$$C_{\vec{k}}(\tau)=\langle n_{\vec{k}}(\vec{r}\,',t)\,n^*_{\vec{k}}(\vec{r}\,',t+\tau)\rangle_t=\langle\sum_{j,l=1}^{N_s^2} e^{-i(\vec{k}.\vec{r}_j(t)-\vec{k}.\vec{r}_l(t+\tau))}\rangle_t$$

When incident wave wavelength is much shorter than the Debye length,

$$k\lambda_D>1$$

even the collective effects due to ion shielding by electron will not affect scattering phase coherence between electrons for such a small wavelength.

For a small scattering wavelength, the cross-correlations terms are negligible :

$$C_{\vec{k}}(\tau)=\langle\sum_{j=1}^{N_s} e^{-i(\vec{k}.\vec{r}_j(t)-\vec{k}.\vec{r}_j(t+\tau))}\rangle_t+\cancel{\langle\sum_{j\neq l}^{N_s(N_s-1)} e^{-i(\vec{k}.\vec{r}_j(t)-\vec{k}.\vec{r}_l(t+\tau))}\rangle_t}$$

For $\tau=0$

$$\langle|n_{\vec{k}}(t)|^2\rangle_t=C_{\vec{k}}(0)=\langle\sum_j^{N_s} e^{i0}\rangle_t=N_s$$

Incoherent Thomson Scattering : correlation and velocity

Scattering signal time correlation only includes electron self scattering :

$$C_{\vec{k}}(\tau)=\langle \sum_{j=1}^{N_s} e^{-i(\vec{k}.\vec{r}_j(t)-\vec{k}.\vec{r}_j(t+\tau))} \rangle_t$$

$$n_{\vec{k}}(t)=\sum_j e^{-i\vec{k}.\vec{r}_j(t)}$$

We assume that for τ delays shorter than $C_{\vec{k}}(\tau)$ correlation time, electron has **constant velocity** trajectory : $\vec{r}_j(t)-\vec{r}_j(t+\tau)=\vec{v}_j(t)\tau$

The velocity still varies at larger time scales.

$$C_{\vec{k}}(\tau)=\langle \sum_{j=1}^{N_s} e^{i\vec{k}.\vec{v}_j(t)\tau} \rangle_t$$

Using **ergodic** hypothesis, we introduce electron velocity distribution : $f_{e\vec{v}}$

$$C_{\vec{k}}(\tau)=N_s \iiint d\vec{v}\, f_{e\vec{v}}(\vec{v})\, e^{i\vec{k}.\vec{v}\tau}$$

The velocity component along $\vec{k}$ only plays a role : $v_k=\vec{v}.\vec{e}_k$

$$\vec{e}_k=\frac{1}{k}\vec{k}$$

f_{ev_k} : probability distribution for electron velocity component along $\vec{k}$

$$C_{\vec{k}}(\tau)=N_s \int d v_k\, f_{ev_k}(v_k)\, e^{i k v_k \tau}$$

Incoherent Thomson Scattering and velocity distribution

$$C_{\vec{k}}(\tau)=N_s\int d v_k f_{e v_k}(v_k)\mathrm{e}^{i k v_k \tau}$$

$$n_{\vec{k}}(t)=\sum_j^{N_s}\mathrm{e}^{-i\vec{k}.\vec{r}_j(t)}$$

The frequency spectrum for a finite power signal :

$$n_{\vec{k}T}(\omega)=\int_t^{t+T} dt\, n_{\vec{k}}(t)\mathrm{e}^{i\omega t}$$

$$N_{\vec{k}}(\omega)=lim_{T\to\infty}\frac{1}{T}\left|n_{\vec{k}T}(\omega)\right|^2$$

The frequency spectrum is the time Fourier transform of the correlation :

$$N_{\vec{k}}(\omega)=\int d\tau\, C_{\vec{k}}(\tau)\mathrm{e}^{i\omega\tau}$$

$$N_{\vec{k}}(\omega)=N_s\int d\tau\int d v_k f_{e v_k}(v_k)\mathrm{e}^{i(\omega+k v_k)\tau}$$

Integration variable substitution : $k\tau\to u$

$$N_{\vec{k}}(\omega)=N_s\frac{2\pi}{k}\int d v_k f_{e v_k}(v_k)\delta\left(\frac{\omega}{k}+v_k\right)$$

$$N_{\vec{k}}(\omega)=N_s\frac{2\pi}{k} f_{e v_k}\left(\frac{-\omega}{k}\right)$$

The Thomson scattering signal spectrum reproduces
the 1D electron velocity distribution along $\vec{k}$.

Knowing the normalization, it is proportional to the **plasma density**.

Incoherent Thomson Scattering : form factor

$$\langle |n_{\vec{k}}(t)|^2 \rangle_t = N_s \qquad n_{\vec{k}}(t) = \sum_j^{N_s} e^{-i\vec{k}.\vec{r}_j(t)}$$

Scalar Form Factor for Incoherent Scattering :

$$S_{\vec{k}} = \frac{1}{N_s} \langle |n_{\vec{k}}(t)|^2 \rangle_t = 1$$

Dynamical Form Factor for Incoherent Scattering :

$$S_{\vec{k}}(\omega) = \frac{1}{N_s} N_{\vec{k}}(\omega)$$

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} f_{e v_k}\left(\frac{-\omega}{k}\right)$$

Dynamical Form Factor for Incoherent Scattering is normalized :

$$\int S_{\vec{k}}(\omega)\, d\omega = 1 \qquad \int f_{e v_k}(v_k)\, dv_k = 1$$

Incoherent Thomson Scattering : scattered power

The scattered power for a scattering volume V_s is the sum of the averaged scattered power for one electron multiplied by the electron number in the volume : $N_s = n_e V_s$

$$\frac{d\sigma}{d\Omega_s}(\theta,\varphi) = \frac{r'^2 I_s}{I_i} = \frac{r'^2 \langle \left| \sum_{j=1}^{N_s} E_{sj} \right|^2 \rangle}{\langle |E_{i0}|^2 \rangle} = \frac{r'^2 \sum_{j=1}^{N_s} \langle |E_{sj}|^2 \rangle}{\langle |E_{i0}|^2 \rangle}$$

$$\frac{d\sigma}{d\Omega_s}(\theta,\varphi) = N_s r_0^2 \cos^2\theta_{pol}$$

The plasma scattered power per unit volume integrated for all directions :

$$\sigma_T = \frac{8\pi}{3} r_0^2 N_s$$

Including the frequency variation :

$$\frac{d\sigma}{d\Omega_s}(\theta,\varphi,\omega) = N_s r_0^2 \cos^2\theta_{pol} \frac{2\pi}{k} f_{e v_k}\left(\frac{-\omega}{k}\right)$$

Incoherent Thomson Scattering : Maxwellian distribution

When the velocity distribution is Maxwellian,

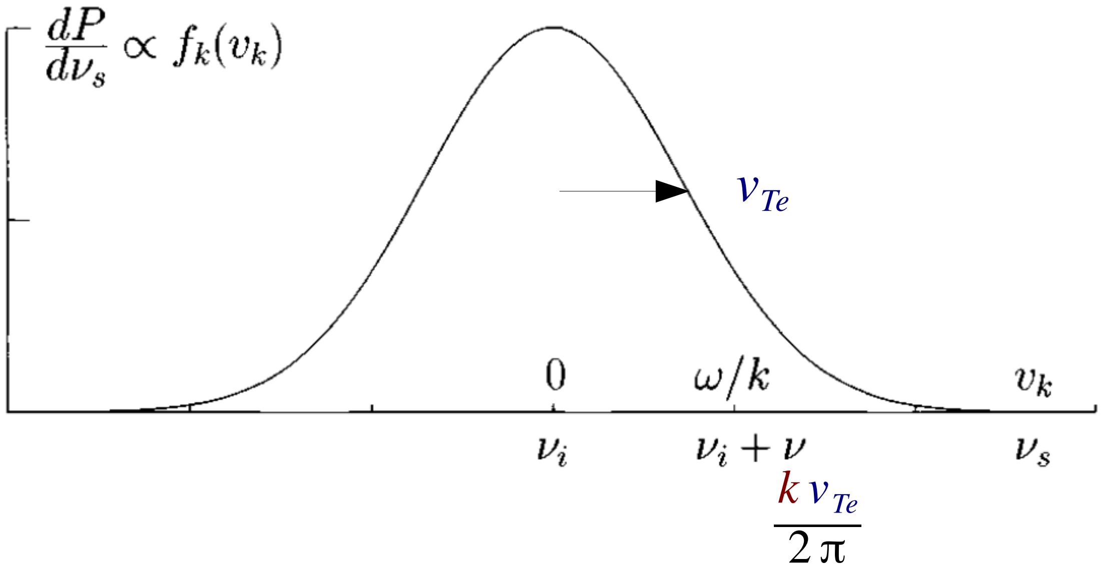
$$f_{v_k e0}(v_k) = \frac{1}{v_{Te}\sqrt{2\pi}} e^{\frac{-v_k^2}{2 v_{Te}^2}} \qquad v_{Te} = \sqrt{\frac{k_B T_e}{m_e}}$$

we can deduce **the electron temperature** from the Gaussian width.

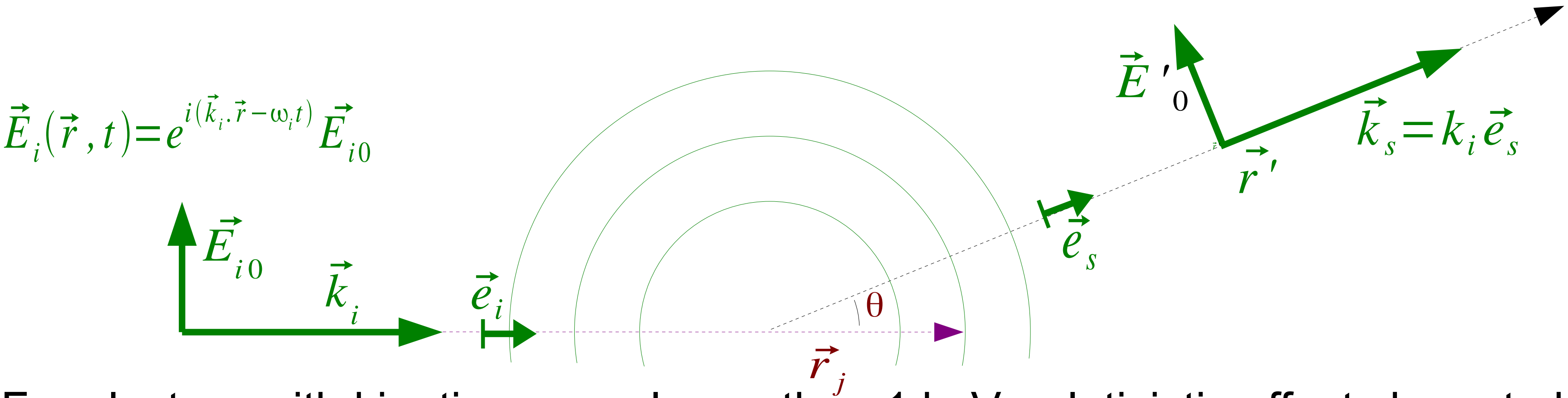
$$S_{\vec{k}}(\omega) = \frac{\sqrt{2\pi}}{k\, v_{Te}} e^{\frac{-\omega^2}{2 k^2 v_{Te}^2}} \qquad \Delta\omega_{e^{-1/2}} = k\, v_{Te}$$

We can also measure the **electron density** :

$$\frac{d\sigma}{d\Omega_s}(\theta,\varphi) = n_e V_s r_0^2 \cos^2\theta_{pol}$$



Thomson Scattering : relativistic acceleration



For electron with kinetic energy larger than 1 keV, relativistic effects have to be taken into account.

$$\vec{\beta}_j(t)=\frac{1}{C}\vec{v}_j(t)$$

Classical electron acceleration by the incident electric field :

$$d_t\vec{\beta}_j=\frac{-q_e}{m_e C}\vec{E}_i(\vec{r}_j(t),t)$$

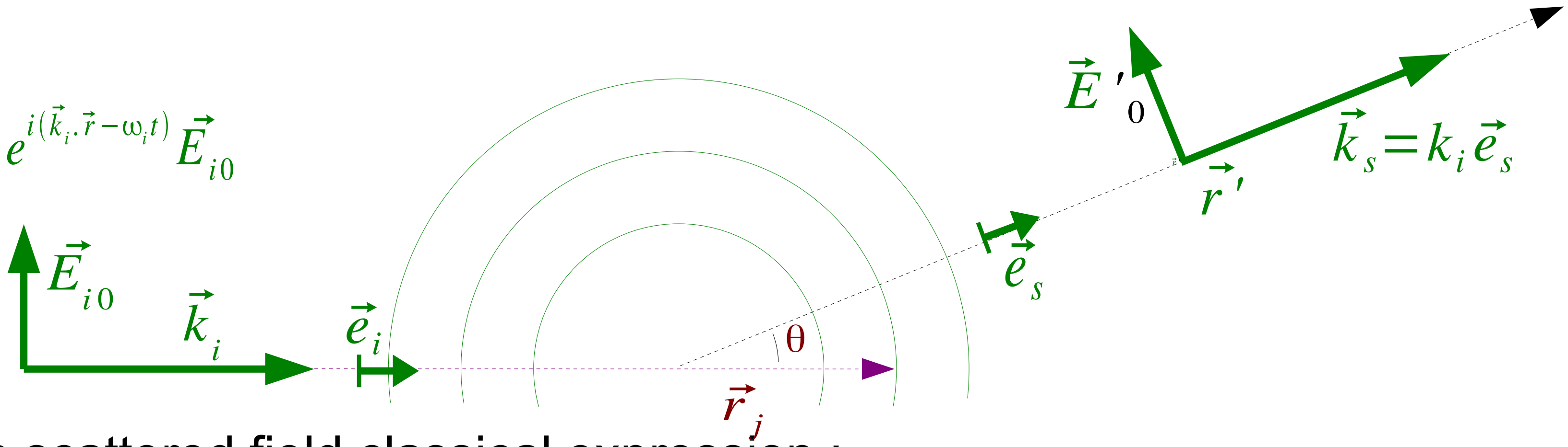
Relativistic electron acceleration :

$$d_t\vec{\beta}_j=\frac{-q_e}{m_e\gamma_j C}\left[\vec{E}_i-(\vec{\beta}_j.\vec{E}_i)\vec{\beta}_j+(\vec{\beta}_j.\vec{E}_i)\vec{e}_i-(\vec{\beta}_j.\vec{e}_i)\vec{E}_i\right]$$

$$\gamma_j=\frac{1}{\sqrt{1-\beta_j^2}}$$

Thomson Scattering : relativistic scattered electric field

$$\vec{E}_i(\vec{r},t)=e^{i(\vec{k}_i.\vec{r}-\omega_i t)}\vec{E}_{i0}$$



Electron scattered field classical expression :

$$\vec{E}_{s\,rel\,j}(\vec{r}\,',t)=\frac{\mu_0 C\, q_e}{4\pi}\frac{\mathrm{e}^{i k_i|\vec{r}\,'-\vec{r}_j|}}{|\vec{r}\,'-\vec{r}_j|}\vec{e}_{sj}\wedge(\vec{e}_{sj}\wedge d_t\vec{\beta}_j)$$

$$\vec{\beta}_j(t)=\frac{1}{C}\vec{v}_j(t)$$

$$\vec{E}_{s\,j}(\vec{r}\,',t)=\frac{\mu_0 q_e^2}{4\pi m_e}\frac{\mathrm{e}^{i k_i|\vec{r}\,'-\vec{r}_j|}}{|\vec{r}\,'-\vec{r}_j|}\mathrm{e}^{i(\vec{k}_i.\vec{r}_j-\omega_i t)}\vec{e}_{sj}\wedge(\vec{e}_{sj}\wedge\vec{E}_{i0})$$

Electron scattered field relativistic expression :

$$\vec{E}_{s\,rel\,j}(\vec{r}\,',t)=\frac{\mu_0 q_e^2}{4\pi m_e}\frac{\mathrm{e}^{i k_i|\vec{r}\,'-\vec{r}_j|}}{|\vec{r}\,'-\vec{r}_j|}\mathrm{e}^{i(\vec{k}_i.\vec{r}_j-\omega_i t)}\frac{1}{(1-\vec{e}_{sj}.\vec{\beta}_j)^3}\vec{e}_{sj}\wedge[(\vec{e}_{sj}-\vec{\beta}_j)\wedge\vec{E}_{i0}]$$

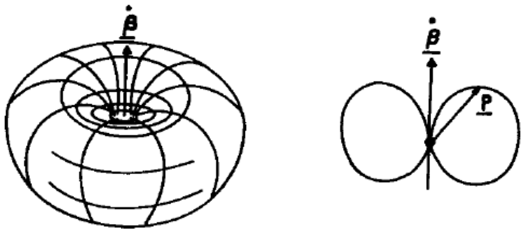
Incoherent Thomson Scattering : relativistic scattered power

Classical scattered cross section power spectrum :

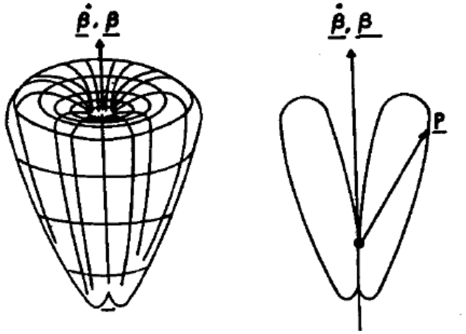
$$\frac{d\sigma}{d\Omega_s}(\theta,\varphi,\omega)=N_s r_0^2 \cos^2\theta_{pol}\frac{2\pi}{k} f_{ev_k}\left(\frac{-\omega}{k}\right)$$

For the relativistic case, the scattering signal frequency spectrum is not proportional to the velocity distribution (1st order development in β):

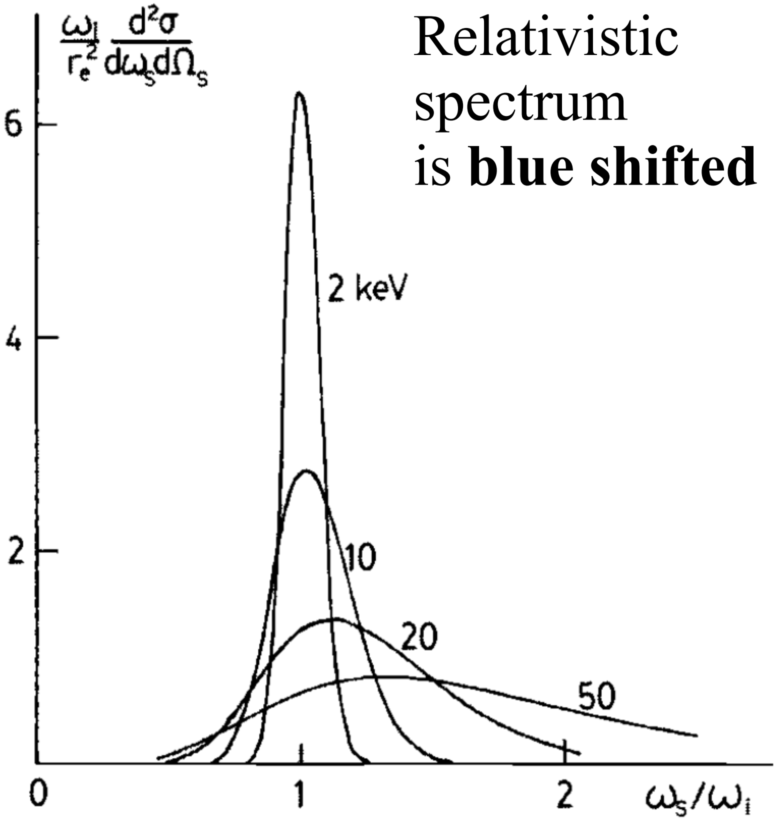
$$\frac{d\sigma_{rel}}{d\Omega_s}(\theta,\varphi,\omega)=N_s r_0^2 \frac{1}{\sqrt{2(1-\cos\theta_{pol})}}\left(1+\frac{3\omega}{2\omega_i}\right)\frac{2\pi}{k_i} f_{ev_k}\left(\frac{-\omega}{k}\right)$$



Classical limit



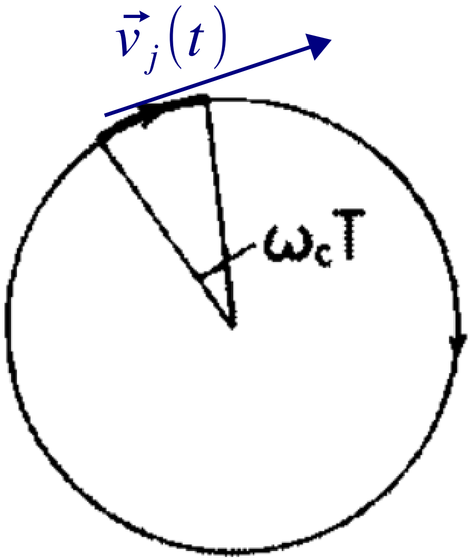
Relativistic case :
enhanced power in the forward direction



Relativistic spectrum is **blue shifted**

Thomson Scattering : magnetized plasma $\omega_i \gg \omega_{ce}$

When $\omega_i \gg \omega_{ce}$ plasma polarization due to the incident wave is not modified by the presence of the magnetic field.
In the presence of the magnetic field, charged particles have cyclotron trajectories.
Charged particle do not have rectilinear trajectories.
But in the case incident wave frequency is much larger than the electron cyclotron frequency, $\omega_i \gg \omega_{ce}$, we will consider that the scattering signal correlation time is much shorter than the electron cyclotron time $T_{C_{\vec{k}}} \omega_{ce} \ll 1$.
The electron then rotates a small angle along the cyclotron circle.
This movement is almost a rectilinear movement.
The angle position along the cyclotron circle is random.



The scattering signal spectrum will then reproduce the velocity distribution.

$$C_{\vec{k}}(\tau) = \left\langle \sum_j^{N_s} e^{i \vec{k} . \vec{v}_j(t) \tau} \right\rangle_t$$

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} f_{e v_k}\left(\frac{-\omega}{k}\right)$$

Thomson Scattering : magnetized plasma $\omega_i \sim \omega_{ce}$

Scattering signal correlation expression for a magnetized plasma

$$C_{\vec{k}}(\tau) = \left\langle \sum_{j=1}^{N_s} e^{-i(\vec{k}.\vec{r}_j(t) - \vec{k}.\vec{r}_j(t+\tau))} \right\rangle_t$$

The phase for each electron is written :

$$\vec{k}.\vec{r}_j(t+\tau) - \vec{k}.\vec{r}_j(t) = \frac{k_\perp u_{j\perp}}{\omega_{ce}} \sin(\omega_{ce}\tau) + k_\parallel v_{j\parallel}\tau$$

$$k_\parallel = \vec{k}.\vec{e}_B \qquad v_{j\parallel} = \vec{v}.\vec{e}_B$$

$$\vec{k}_\perp = \vec{k} - k_\parallel \vec{e}_B \qquad \vec{v}_{j\perp} = \vec{v} - v_{j\parallel}\vec{e}_B$$

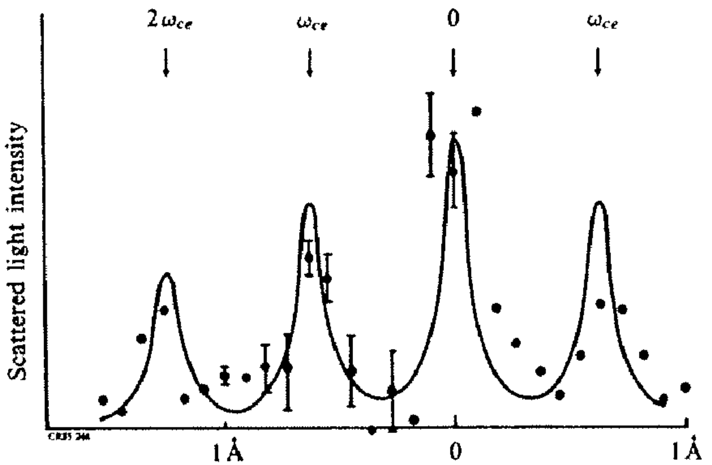
We use 1st type Bessel functions to develop sinus function inside the phase :

$$e^{iz\sin\theta} = \sum_{n=-\infty}^{\infty} J_n(z) e^{in\theta}$$

The electron correlation phase is written :

$$e^{i\left[\frac{k_\perp u_{j\perp}}{\omega_{c\alpha}} \sin(\omega_{c\alpha}\tau)\right]} = \sum_{n=-\infty}^{\infty} J_n\left(\frac{k_\perp u_{j\perp}}{\omega_{c\alpha}}\right) e^{in\omega_{c\alpha}\tau}$$

$$C_{\vec{k}}(\tau) = \left\langle \sum_{j=1}^{N_s} e^{ik_\parallel v_{j\parallel}\tau} \sum_{n=-\infty}^{+\infty} J_n\left(\frac{k_\perp u_{j\perp}}{\omega_{ce}}\right) e^{in\omega_{ce}\tau} \right\rangle_t$$



P Carolan & D Evans,
Int. Conf. Ph. Ion. Gases (1971)

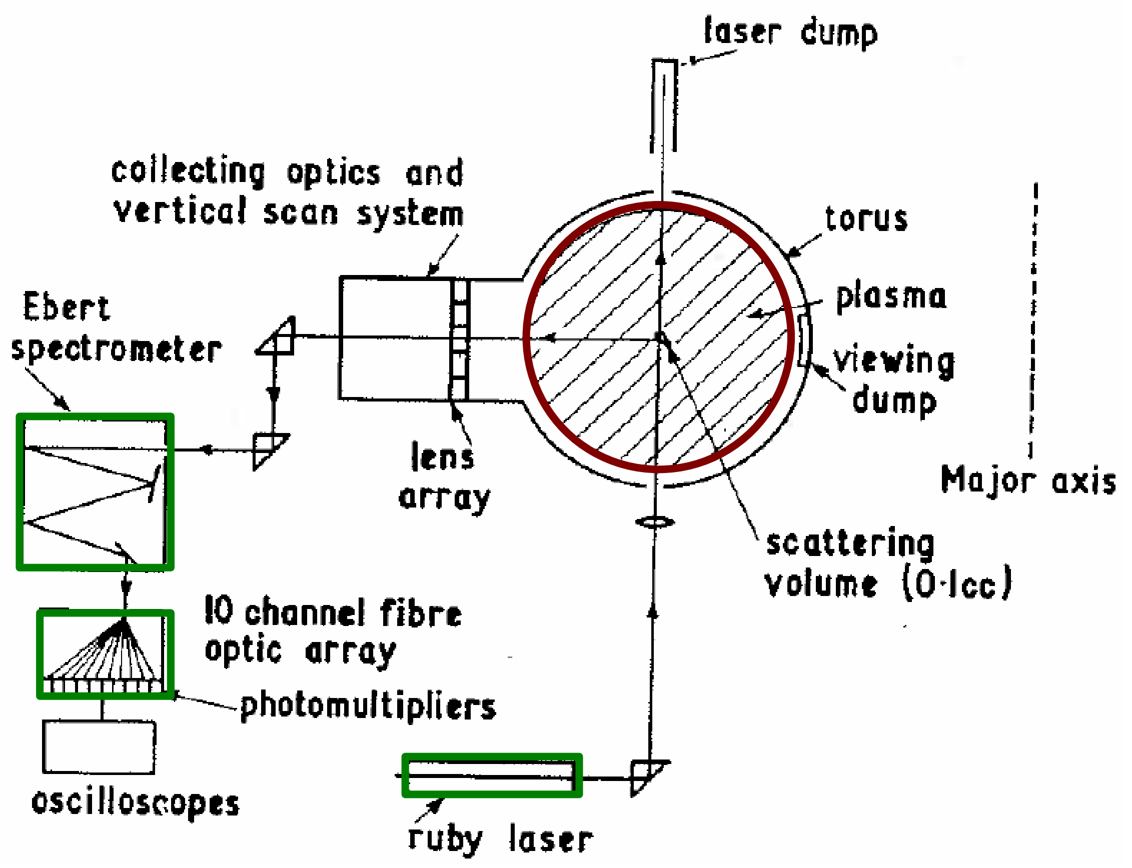
$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} \sum_{n=-\infty}^{+\infty} \iiint d^3\vec{v}\, J_n\left(\frac{k_\perp v_\perp}{\omega_{ce}}\right) \delta(\omega + k_\parallel v_\parallel + n\omega_{ce}) f_{e\vec{v}}(\vec{v})$$

Incoherent Thomson Scattering on T3 Tokamak

For first Tokamaks (T1, T3), plasma electron temperature was evaluated using the magnetic measurements and using models connecting the plasma conductivity to the plasma collisionality, and then the electron temperature.

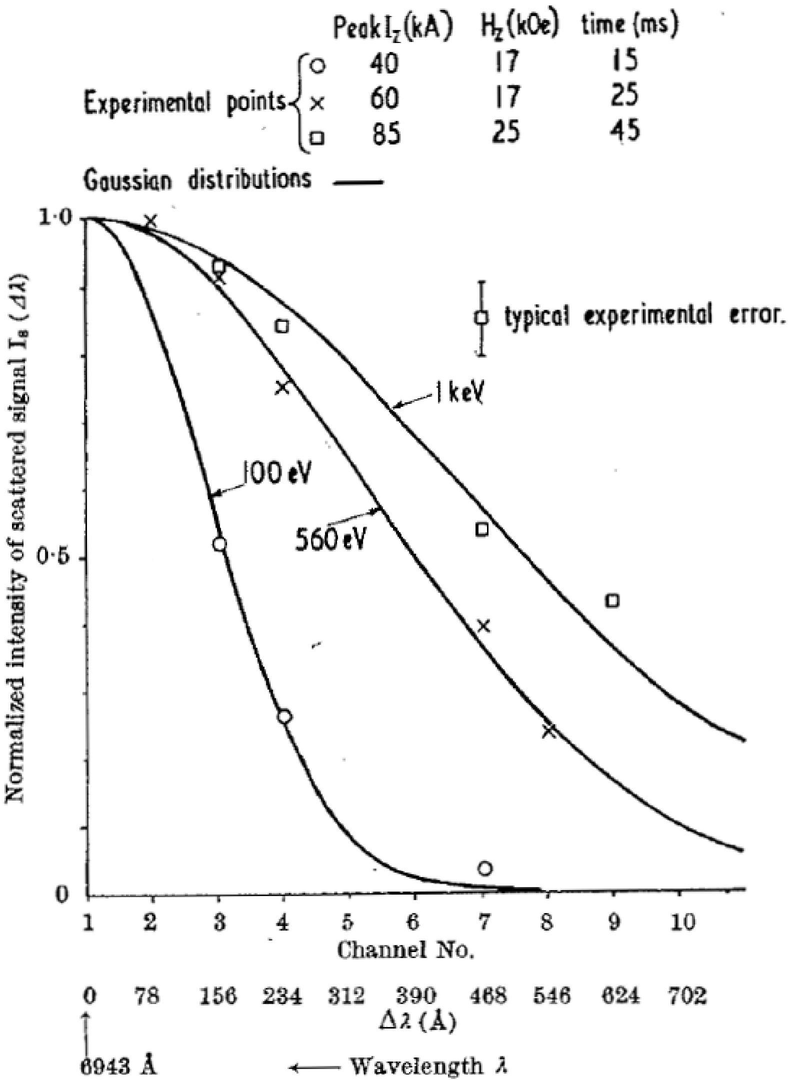
In order to check this evaluation, a British group made Thomson scattering measurements on the Tokamak Plasma.

Ruby laser : $\lambda_i = 0.6943\,\mu m$



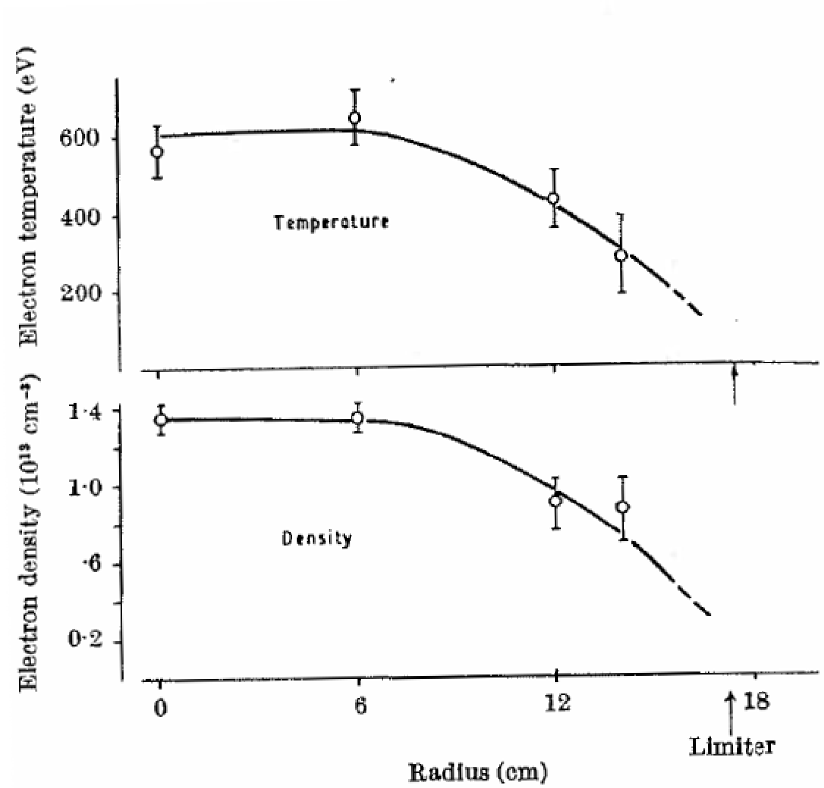
Arstimovich et al., Plasma Physics, **7** (1965), p 305
Peacock et al., Nature, **224**, (1/11/1969), p 488

T3 Tokamak electron density and temperature profiles



Scattered power was measured for 10 different frequency shift.

Data are fitted with Maxwellian in order to estimate the temperature.



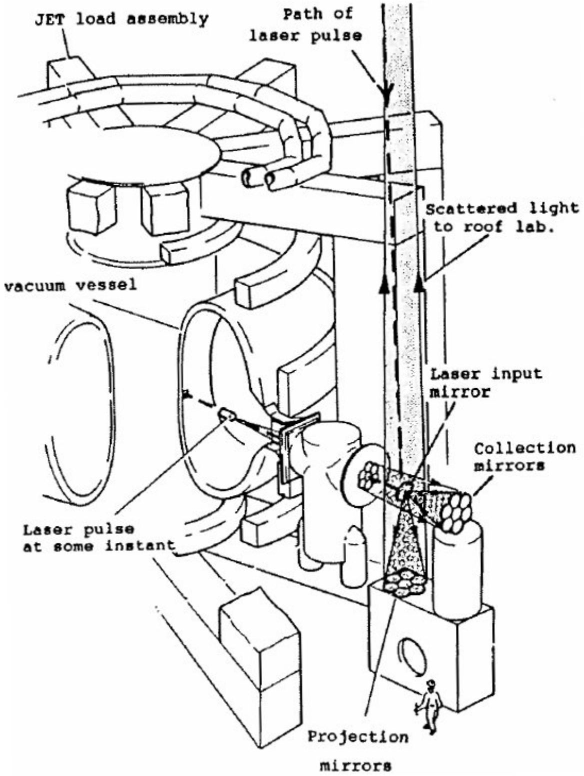
$$\Delta \nu_i = (-\nu_i / \lambda_i)\, \Delta \lambda_i \qquad \Delta \nu = v_e / \lambda$$

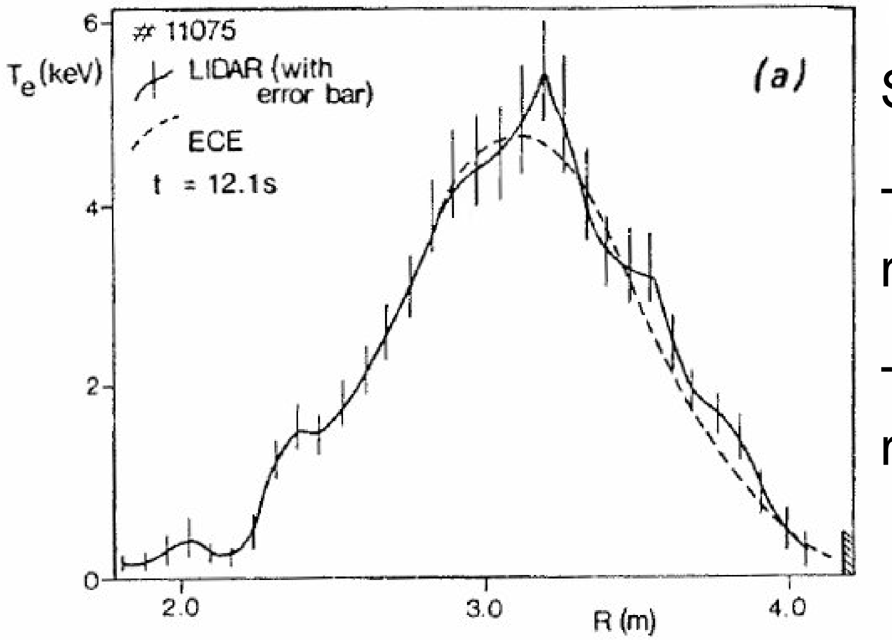
$$\nu_i \lambda_i = C = (\nu_i + \Delta \nu_i)(\lambda_i + \Delta \lambda_i)$$

Measurements were done at different positions in order to get the electron temperature profile.

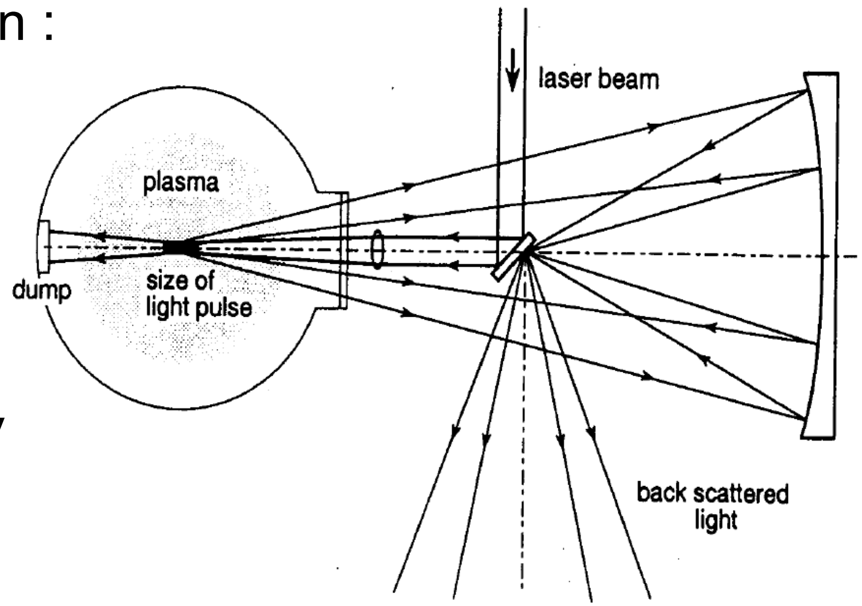
LIDAR Incoherent TS on JET

LIght Detection And Ranging : short time laser pulse back-scattering measurements. Same measurement principle as radars : localization is function of the time of flight.
The LIDAR only needs one **access port to the device**.
Spatial resolution is proportional to the Laser pulse short duration : we need a more sensitive detector.
A Thomson scattering LIDAR is present on JET :
Pulsed Ruby Laser (694 nm, 2 J, 220 ps, 0,5 Hz)
Polychromator with 6 spectral channels
MCP Photomultiplier



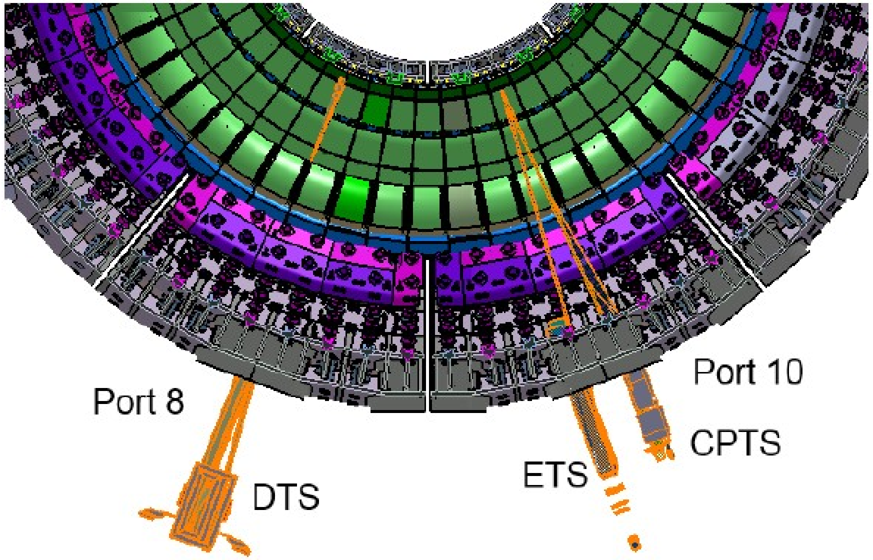


Spatial resolution :
15 cm
Temperature resolution :
± 5 to 20%
Temperature range :
0.5 to 20 keV

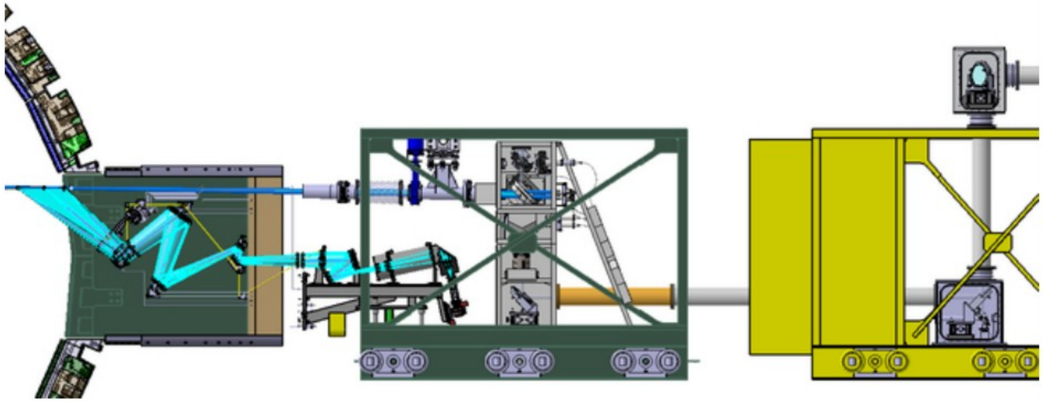


Salzmann et al., Nuclear Fusion, **27**, p 1925 (1987)

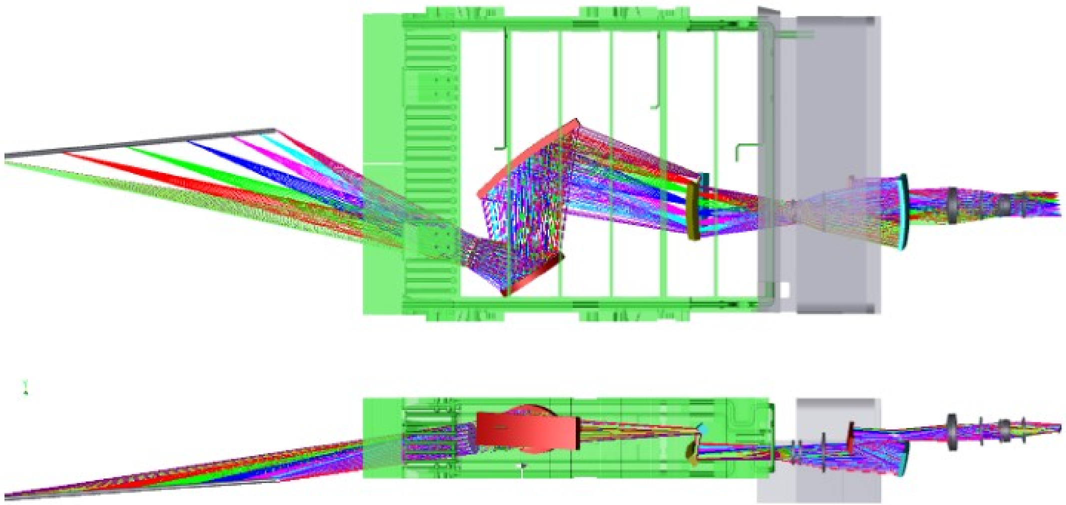
ITER incoherent Thomson Scattering



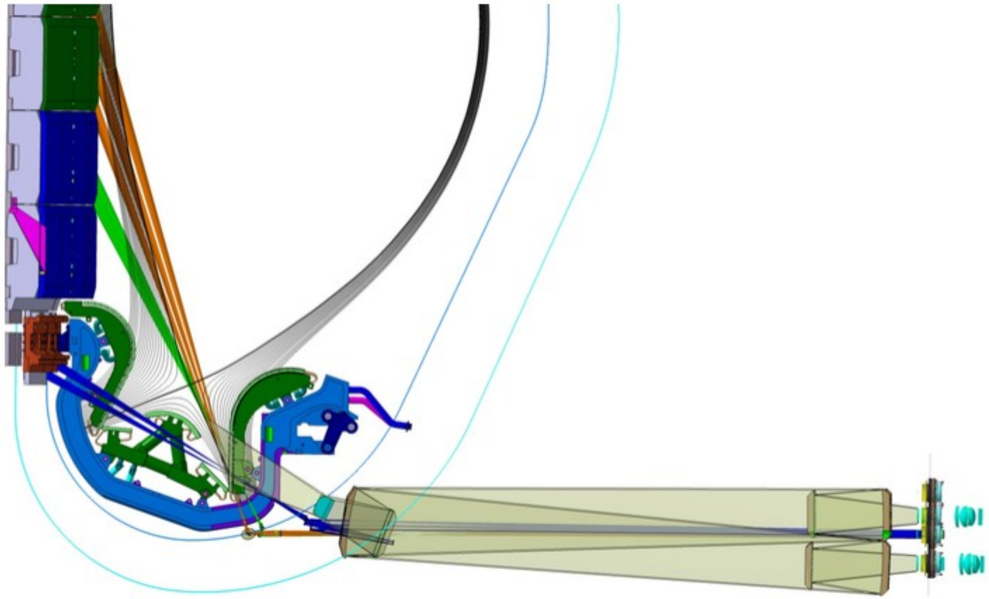
3 Thomson Scattering systems around ITER



Edge Thomson Scattering system



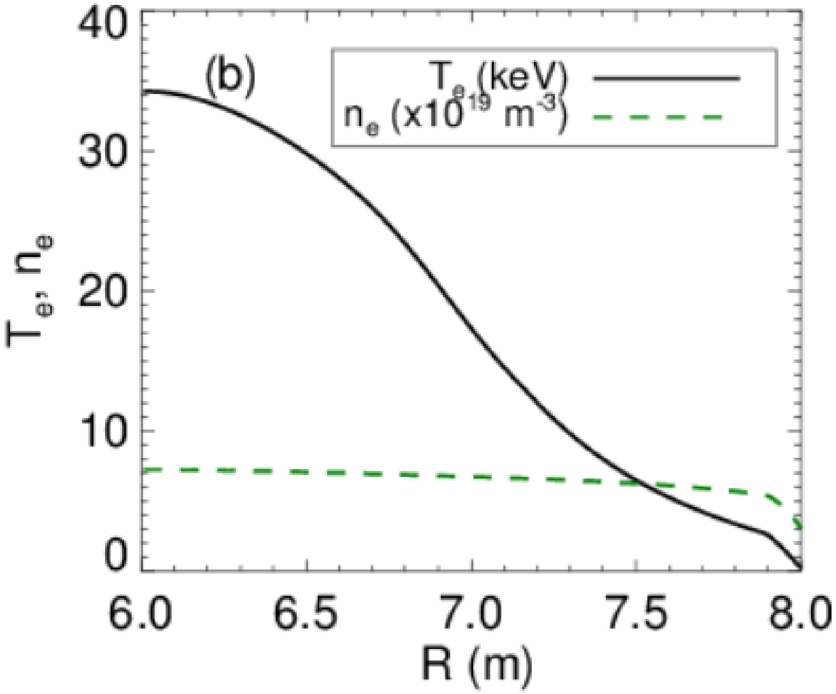
Core Plasma Thomson Scattering system

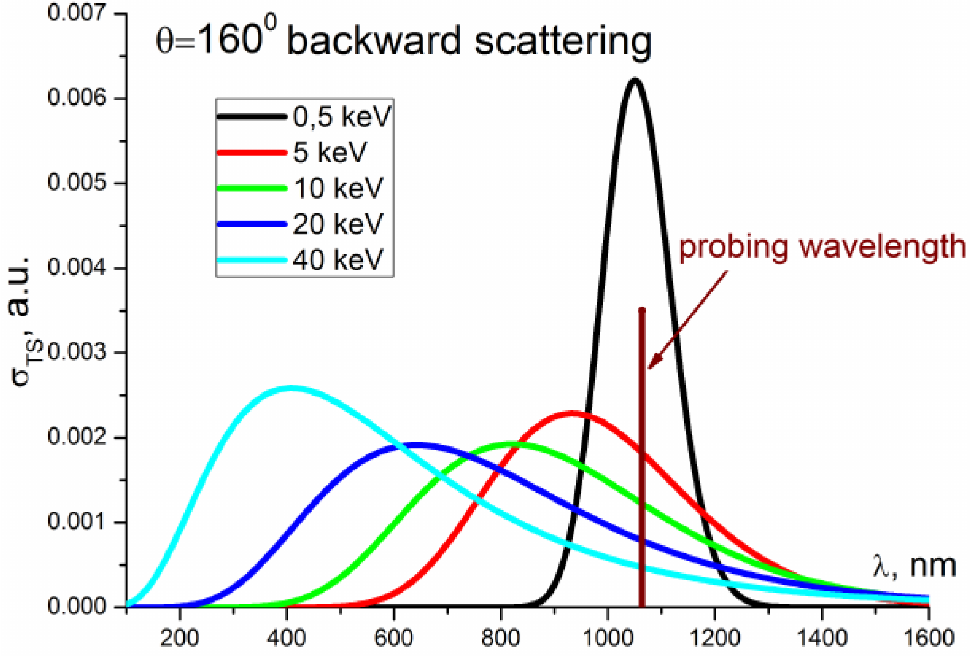


Divertor Thomson Scattering system

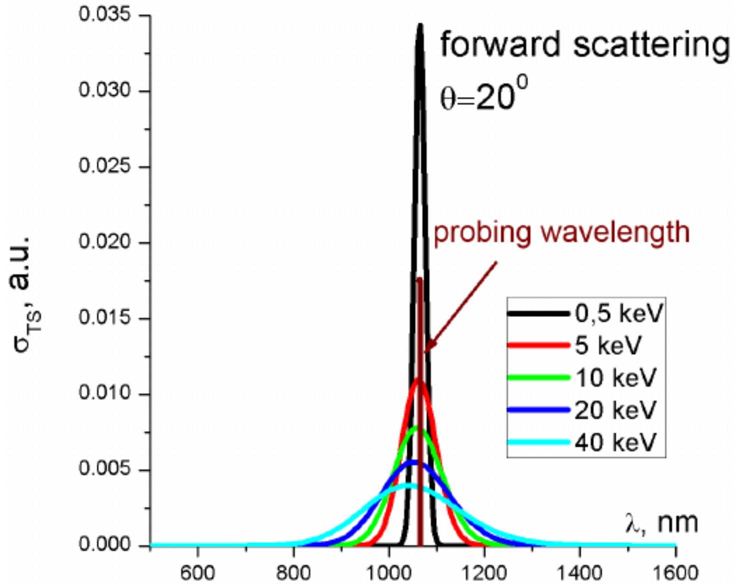
Bassan et al., J. Instr., **11**, C01052 (2016)

ITER incoherent Thomson Scattering

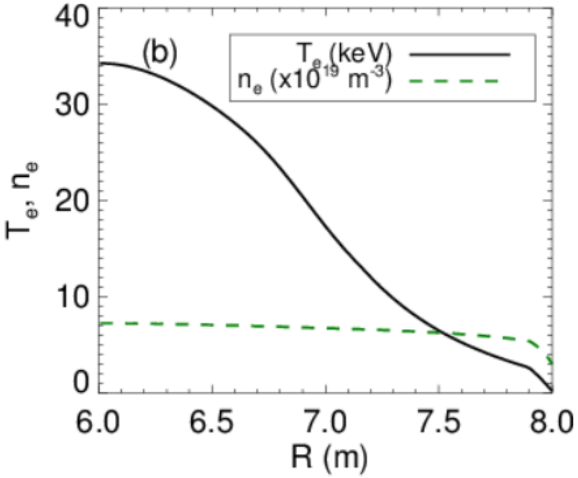




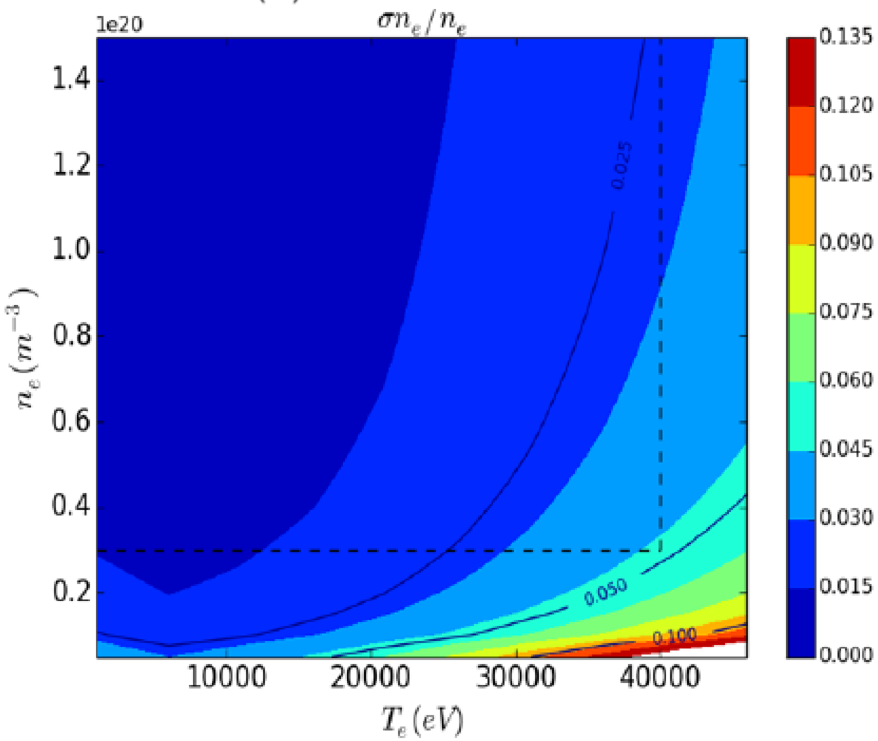
Simulated Thomson scattering cross-section depending on the scattering direction and electron temperature.

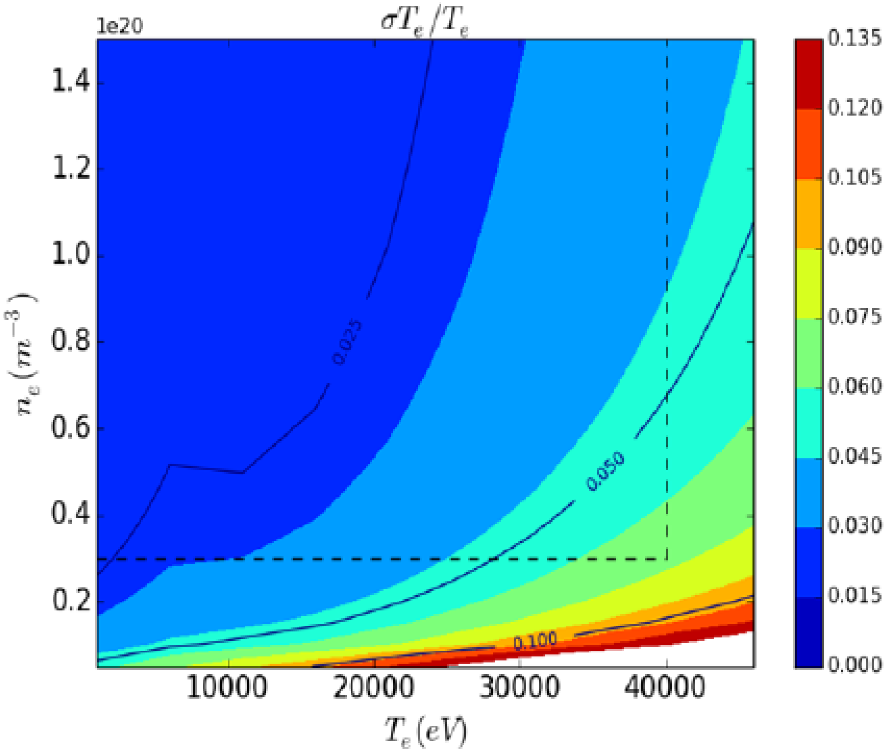


ITER incoherent Thomson Scattering



Electron density and temperature measurement accuracy depends on their value in the plasma





Scannell et al., J. Instr., **12**, C11010 (2017)

Coherent Thomson Scattering

Coherent Thomson Scattering

The Scattering Signal is the sum of all scatterer scattering phases :

$$n_{\vec{k}}(t)=\sum_{j}^{N_s} \mathrm{e}^{-i\vec{k}.\vec{r}_j(t)}$$

The sum can be replaced with the Klimontovich description of the density :

$$n_{Kl}(\vec{r},t)=\sum_j \delta(\vec{r}-\vec{r}_j(t))$$

$$n_{\vec{k}}(t)=\iiint_{Vs} n_{Kl}(\vec{r},t)\mathrm{e}^{-i\vec{k}.\vec{r}} d^3\vec{r}$$

When the scattering wavelength $\lambda=2\pi/k$ is large enough, $k\lambda_D<1$ $\lambda>2\pi\lambda_D$
we switch from the microscopic description to a mesoscopic description :

$$n_{Kl}(\vec{r}\,t)\rightarrow n_e(\vec{r}\,t)$$

For a mesoscopic point of view, the scattering signal is the spatial Fourier transform of the electron density :

$$n_{\vec{k}}(t)=\iiint_{Vs} n_e(\vec{r},t)\mathrm{e}^{-i\vec{k}.\vec{r}} d^3\vec{r}$$

Coherent Thomson Scattering

Because of the electron mass, plasma **electrons** are the main **scatterer** particles for Thomson scattering.

But for scattering wavelength larger than the Debye length ($k\lambda_D<1$),
the **electron dynamics** are affected by **ion dynamics** due to the screening by electrons.

Electron density fluctuations are linked also to ion density fluctuations.

The plasma will be considered as a **dielectric medium**.

Plasma susceptibility

Expression of the **electric displacement** for a dielectric medium :
the electric field is modified by the presence of **dipoles** :

$$\vec{D}=\varepsilon_r\varepsilon_0\vec{E}=\varepsilon_0\vec{E}+\vec{P}$$

The dipole field is function of the plasma **susceptibility** :

$$\vec{P}=\chi\varepsilon_0\vec{E}$$

The plasma response is the sum of the electron and the ion response :

$$\vec{P}=\vec{P}_e+\vec{P}_i \qquad \vec{P}_e=\chi_e\varepsilon_0\vec{E} \qquad \vec{P}_i=\chi_i\varepsilon_0\vec{E}$$

$$\vec{D}=\varepsilon_0(1+\chi_e+\chi_i)\vec{E} \qquad \varepsilon_r=1+\chi_e+\chi_i$$

The charge associated for electrons due to the electric displacement :

$$\rho_e=-\vec{\nabla}.\vec{P}_e=\frac{-\chi_e}{1+\chi_e+\chi_i}\vec{\nabla}.\vec{D}$$

We have to determine $\vec{D}$ in order to know the electron density fluctuations used for scattering signal :

$$n_e=\frac{-1}{q_e}\rho_e$$

Coherent Thomson Scattering: shielding effect

For any **test particle** with charge q_n and velocity $\vec{v}_n$,
The charge dynamics is :

$$\rho_n(\vec{x},t)=q_n\delta(\vec{x}-\vec{v}_n t)$$

The Fourier transform of this moving charge is :

$$\rho_n(\vec{k},\omega)=2\pi q_n\delta(\vec{k}.\vec{v}_n+\omega)$$

$$\rho_n(\vec{k},\omega)=\iiint d^3\vec{r}\int dt\,\rho_n(\vec{r},t)\mathrm{e}^{i(\omega t-\vec{k}.\vec{r})}$$

This test particle generates an **electric displacement** field :

$$\rho_n=\vec{\nabla}.\vec{D}$$

So the test particle modifies the electron charge distribution through the **screening effect** :

$$\rho_e(\vec{k},\omega)=\frac{-\chi_e}{1+\chi_e+\chi_i}2\pi q_n\delta(\vec{k}.\vec{v}_n+\omega)$$

$$\rho_e=\frac{-\chi_e}{1+\chi_e+\chi_i}\vec{\nabla}.\vec{D}$$

The electron charge distribution, will be the sum of the electron distribution,
the electron shielding electron distribution and the electron shielding ion distribution
The resulting electric charge distribution is modified by all test particles :

$$\tilde{\rho}_e(\vec{k},\omega)=\sum_n^{N_{part}}\frac{-\chi_e}{1+\chi_e+\chi_i}2\pi q_n\delta(\vec{k}.\vec{v}_n+\omega)$$

$$n_e(\vec{k},\omega)=\frac{-1}{q_e}\rho_e(\vec{k},\omega)$$

Z_i: ion charge number

$$n_e(\vec{k},\omega)=\frac{-1}{q_e}\sum_j^{N_{el}}\left[1-\frac{\chi_e}{1+\chi_e+\chi_i}\right]2\pi(-q_e)\delta(\vec{k}.\vec{v}_j+\omega)+\frac{-1}{q_e}\sum_l^{N_{ion}}\left[\frac{-\chi_e}{1+\chi_e+\chi_i}\right]2\pi Z_i q_e\delta(\vec{k}.\vec{v}_l+\omega)$$

Coherent Thomson Scattering: the form factor

Electron distribution corrected by the shielding effect :

$$n_e(\vec{k},\omega)=\frac{-1}{q_e}\sum_j^{N_{el}}\left[1-\frac{\chi_e}{1+\chi_e+\chi_i}\right]2\pi(-q_e)\delta(\vec{k}.\vec{v}_j+\omega)+\frac{-1}{q_e}\sum_l^{N_{ion}}\left[\frac{-\chi_e}{1+\chi_e+\chi_i}\right]2\pi Z_i q_e\delta(\vec{k}.\vec{v}_l+\omega)$$

Dynamical form factor :

$$S_{\vec{k}}(\omega)=\frac{1}{T N_s}\left|n_{eT}(\vec{k},\omega)\right|^2$$

$$S_{\vec{k}}(\omega)=\frac{2\pi}{N_s}\left\langle\sum_j^{N_{el}}\left[1-\frac{\chi_e}{1+\chi_e+\chi_i}\right]^2\delta(\vec{k}.\vec{v}_j+\omega)+\sum_l^{N_{ion}}\left[\frac{-\chi_e}{1+\chi_e+\chi_i}\right]^2 Z_l^2\delta(\vec{k}.\vec{v}_l+\omega)\right\rangle_T$$

For each species ι, $F_{\vec{v}\iota}$ is the velocity distribution :

$$\left\langle\sum_j^{N_\iota}\delta(\vec{k}.\vec{v}_j+\omega)\right\rangle_T=\iiint_{V_s}d^3\vec{r}\iiint d^2\vec{v}\,F_{\vec{v}\iota}(\vec{v})\delta(\vec{k}.\vec{v}+\omega)$$

$f_{v_k\iota}$ is the 1D velocity distribution along $\vec{k}$

$$\left\langle\sum_j^{N_\iota}\delta(\vec{k}.\vec{v}_j+\omega)\right\rangle_T=V_s\int dv_k\,f_{v_k\iota}(v_k)\delta(k\,v_k+\omega)$$

$$\left\langle\sum_j^{N_\iota}\delta(\vec{k}.\vec{v}_j+\omega)\right\rangle_T=V_s\frac{1}{k}f_{v_k\iota}\left(\frac{-\omega}{k}\right)$$

The form factor expression :

$$S_{\vec{k}}(\omega)=\frac{2\pi}{k}\left[\left|1-\frac{\chi_e}{1+\chi_e+\chi_i}\right|^2 f_{v_k e}\left(\frac{-\omega}{k}\right)+\left|\frac{-\chi_e}{1+\chi_e+\chi_i}\right|^2 Z_i^2 f_{v_k i}\left(\frac{-\omega}{k}\right)\right]$$

Coherent Thomson Scattering: susceptibility

The form factor expression :

$$S_{\vec{k}}(\omega)=\frac{2\pi}{k}\left[\left|1-\frac{\chi_e}{1+\chi_e+\chi_i}\right|^2 f_{v_k e}\left(\frac{-\omega}{k}\right)+\left|\frac{-\chi_e}{1+\chi_e+\chi_i}\right|^2 Z_i^2 f_{v_k i}\left(\frac{-\omega}{k}\right)\right] \qquad k\lambda_D<1$$

The dynamical form factor does not depend only on electron velocity distribution, but also on **each ion species velocity distribution**.

When we neglect the electron screening for ions and other electrons, $\chi_e=0$ and all $\chi_i=0$ we obtain the expression for **incoherent** Thomson scattering :

$$S_{\vec{k}}(\omega)=\frac{2\pi}{k} f_{v_k e}\left(\frac{-\omega}{k}\right)$$

What are the **values** of χ_e and χ_i ?

For cold plasmas longitudinal waves :

$$\varepsilon_r=1-\frac{\omega_p^2}{\omega^2} \qquad \varepsilon_r=1+\chi \qquad \chi=-\frac{\omega_p^2}{\omega^2}$$

These expressions apply to the incident beam characteristics $\vec{k}_i, \omega_i$
They do not apply for waves corresponding to thermal plasma particle velocities. $\vec{k}, \omega_D=k\,v_{Te}$
We need to apply kinetic theory : the wave electric field modifies the each species velocity distribution.

Coherent Thomson Scattering: susceptibility

Using Vlasov equation :

$$\partial_t f_\iota + \vec{v}.\vec{\nabla}_{\vec{r}} f_\iota + \frac{q_\iota}{m_\iota}\vec{E}.\vec{\nabla}_{\vec{v}} f_\iota = 0$$

$f_{\iota 0}$ is the initial particle position and velocity distribution.

$f_{\iota 1}(\vec{k},\omega)$ is the response to an external Fourier mode perturbation : $\vec{E}_1(\vec{k},\omega)$

The Fourier transform of the 1st order Vlasov equation is :

$$-i\omega f_{\iota 1} + i\vec{k}.\vec{v} f_{\iota 1} + \frac{q_\iota}{m_\iota}\vec{E}_1.\vec{\nabla}_{\vec{v}} f_{\iota 0} = 0$$

$$f_{\iota 1} = \frac{q_\iota}{i m_\iota}\frac{\vec{E}_1.\vec{\nabla}_{\vec{v}} f_{\iota 0}}{\omega - \vec{k}.\vec{v}}$$

A electric current $\vec{j}_{\iota 1}$ is created by the $\vec{E}_1$ electric field :

$$\vec{j}_{\iota 1} = \iiint q_\iota \vec{v} f_{\iota 1} d^3\vec{v}$$

$$\vec{j}_{\iota 1} = \frac{q_\iota^2}{i m_\iota}\iiint \frac{\vec{v}(\vec{E}_1.\vec{\nabla}_{\vec{v}} f_{\iota 0})}{\omega - \vec{k}.\vec{v}} d^3\vec{v}$$

$$\vec{j}_{\iota 1} = \partial_t \vec{P}_{\iota 1} \qquad \vec{P}_{\iota 1} = \epsilon_0 \chi_\iota . \vec{E}_1 \qquad \Rightarrow \qquad \vec{j}_{\iota 1} = -i\omega\epsilon_0\chi_\iota . \vec{E}_1$$

The **conductivity** tensor connect the electric field to the current : $\vec{j}_{\iota 1} = \bar{\bar{\sigma}}_\iota . \vec{E}_1$

We integrate the velocity distribution for the 2 directions perpendicular to $\vec{k}$

We assume the initial distribution is **isotropic** (*no external magnetic field*).

The species **susceptibility** is connected to their conductivity : $\chi_\iota = -\frac{\sigma_\iota}{i\omega\epsilon_0}$

$$\chi_\iota(\vec{k},\omega) = \frac{q_\iota^2}{m_\iota \epsilon_0 k}\int \frac{\partial_v f_{\iota k 0}}{\omega - k v_k} d v_k$$

Coherent Thomson Scattering : Maxwellian distribution

Susceptibility for each species :

$$\chi_\iota = \frac{q_\iota^2}{m_\iota \epsilon_0 k} \int \frac{\partial_v f_{v_k \iota 0}}{\omega - k v_k} d v_k$$

This is a Landau damping like improper integral

For a Maxwellian distribution :

$$f_{v_k \iota 0}(v_k) = n_{\iota 0} \frac{1}{\sqrt{2\pi}} \left(\frac{1}{v_{T\iota}}\right) e^{\frac{-v_k^2}{2 v_{T\iota}^2}} \qquad v_{T\iota} = \sqrt{\frac{k_B T_\iota}{m_\iota}}$$

According to Fried & Comte :

$$\lambda_{D\iota} = \sqrt{\frac{\varepsilon_0 k_B T_\iota}{n_\iota q_\iota^2}} = \frac{v_{T\iota}}{\omega_{p\iota}}$$

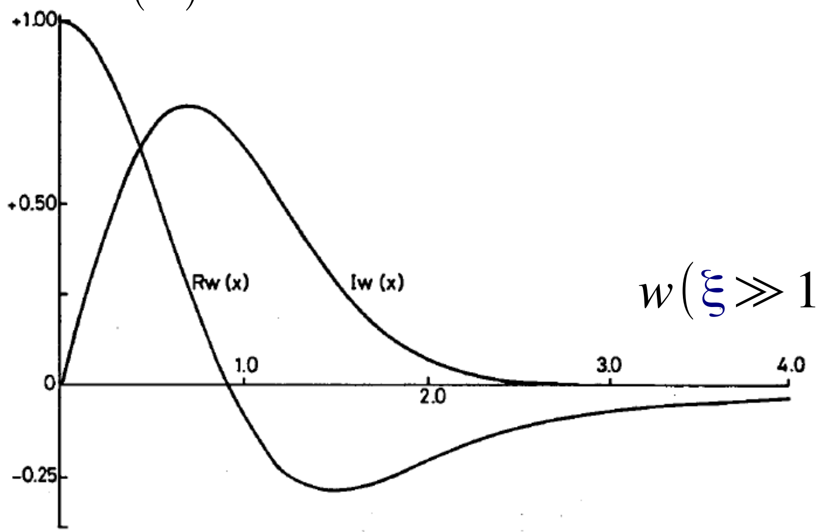
$$\chi_\iota = \frac{1}{k^2 \lambda_{D\iota}^2} w\left(\frac{\omega}{k v_{T\iota} \sqrt{2}}\right) \qquad \xi_\iota = \frac{\omega}{k v_{T\iota} \sqrt{2}}$$

w is the plasma **dispersion function** :

$$w(\xi) = 1 - 2\xi e^{-\xi^2} \int_0^\xi e^{\zeta^2} d\zeta + i\sqrt{\pi}\,\xi e^{-\xi^2}$$

The complex part of the dispersion function corresponds to the Landau Damping.

$w(0)=1$



$w(\xi \gg 1) \sim 0$

Coherent Thomson Scattering : Salpeter approximation

Susceptibility for each species :

$$\chi_\iota = \frac{1}{k^2 \lambda_{D\iota}^2} w(\xi_\iota) \qquad \xi_\iota = \frac{\omega}{k\, v_{T\iota} \sqrt{2}} \qquad w(\xi) = 1 - 2\xi e^{-\xi^2} \int_0^\xi e^{\zeta^2} d\zeta + i\sqrt{\pi}\,\xi e^{-\xi^2}$$

Since $m_e \ll m_i$ and for most cases $T_e \sim T_i$, electron Doppler frequencies are much larger than the ion Doppler frequencies : $k\, v_{Ti} \ll k\, v_{Te}$

For ω in the range of electron Doppler frequencies : $\boxed{\omega \sim k\, v_{Te} \sqrt{2}}$

$$\xi_i = \frac{\omega}{k\, v_{Ti} \sqrt{2}} \gg 1$$

Limit for the plasma dispersion relation:

$$w(\xi_i) \ll 1$$

Ion susceptibility is weak:

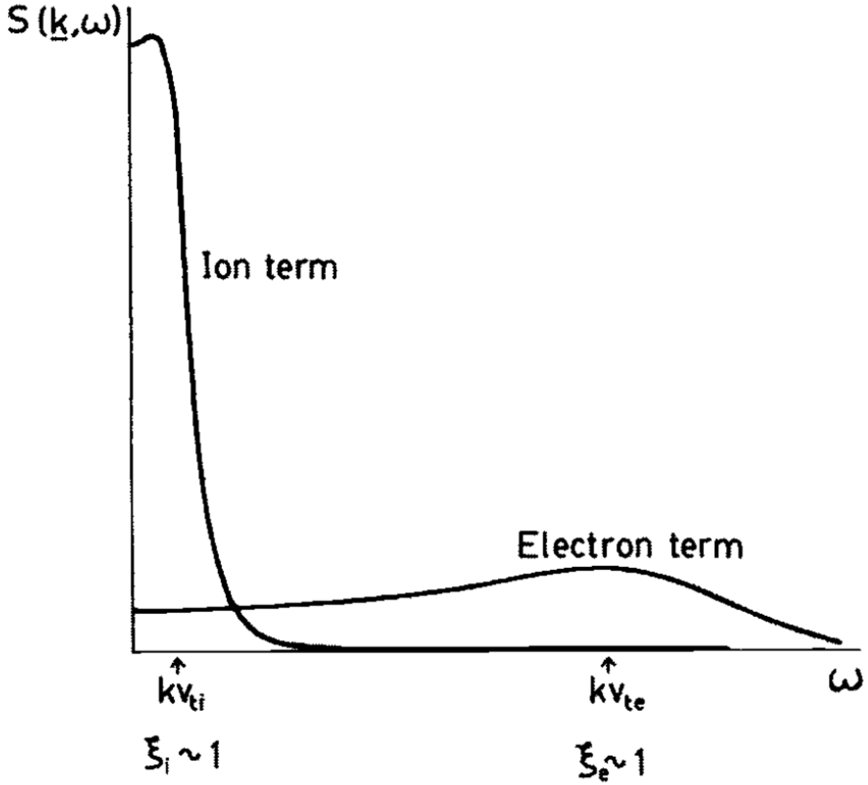
$$\chi_i \ll 1$$

Ion velocity distribution factor is small :

$$f_{v_k i}\left(\frac{-\omega}{k}\right) \propto e^{-\xi_i^2} \ll 1$$

The electron term is dominant :

$$\boxed{S_{\vec{k}}(\omega) \sim \frac{2\pi}{k} \left| \frac{1}{1+\chi_e} \right|^2 f_{v_k e}\left(\frac{-\omega}{k}\right)}$$



Coherent Thomson Scattering : Salpeter approximation

Susceptibility for each species :

$$\chi_\iota = \frac{1}{k^2 \lambda_{D\iota}^2} w(\xi_\iota) \qquad \xi_\iota = \frac{\omega}{k\, v_{T\iota} \sqrt{2}} \qquad w(\xi) = 1 - 2\xi\, e^{-\xi^2} \int_0^\xi e^{(\zeta^2)} d\zeta + i\sqrt{\pi}\, \xi\, e^{-xi^2}$$

For ω in the range of ion Doppler frequencies : $\omega \sim k\, v_{Ti} \sqrt{2}$

For the electrons : $\xi_e \ll 1$

The dispersion function factor is close to 1:

$$w(\xi_e) \sim 1$$

The electron susceptibility is almost constant:

$$\chi_e = \alpha^2 \qquad \alpha = \frac{1}{k \lambda_D}$$

For the ions : $\xi_i \sim 1$

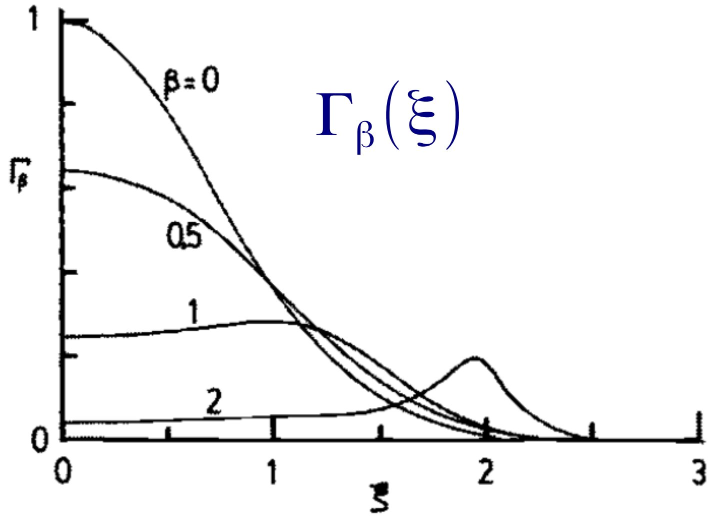
$$\beta^2 = \frac{1}{k^2 \lambda_D^2} \frac{T_e}{T_i}$$

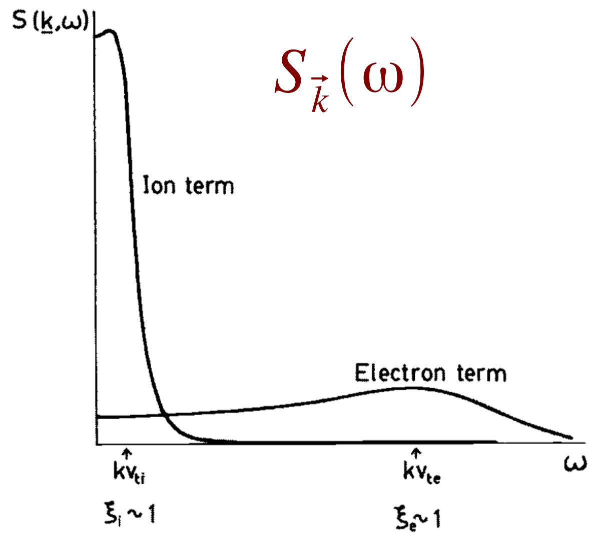
We define the Salpeter function :

$$\Gamma_\beta(\xi) = \frac{e^{-\xi^2}}{|1 + \beta^2 w(\xi)|^2}$$

The form factor includes electron and ion terms :

$$S_{\vec{k}}(\omega) \sim \frac{\sqrt{2\pi}}{k\, v_{Te}} \left(\frac{1}{1+\alpha^2}\right)^2 e^{-\xi_e^2} + \frac{\sqrt{2\pi}}{k\, v_{Ti}} Z_i^2 \left(\frac{\alpha^2}{1+\alpha^2}\right)^2 \Gamma_\beta(\xi_i)$$





Coherent Thomson Scattering : practical application

Coherent Thomson scattering needs the condition : $k\lambda_D < 1$

Salpeter approximation :

$$\xi_\iota = \frac{\omega}{k\, v_{T\iota}\sqrt{2}}$$

$$S_{\vec{k}}(\omega) = \frac{\sqrt{2\pi}}{k\, v_{Te}}\Gamma_\alpha(\xi_e) + \frac{\sqrt{2\pi}}{k\, v_{Ti}} Z_i \left(\frac{\alpha^2}{1+\alpha^2}\right)^2 \Gamma_\beta(\xi_i)$$

$$\Gamma_\beta(\xi) = \frac{e^{-\xi^2}}{\left|1+\beta^2 w(\xi)\right|^2}$$

The scattering dynamical form factor does not reproduce directly the velocity distribution : the ion temperature estimation needs model fitting on measurements from multiple spectral channels.

We need to use small angle forward scattering or long wavelength incident beam.

Small angle forward scattering :

- difficult to separate scattered field from incident beam.

Long wavelength sources :

- long wavelength laser
- microwave sources

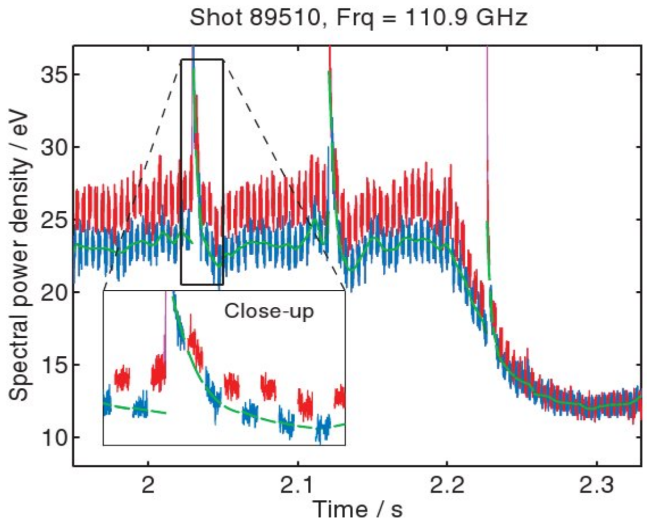
Multiple ions species might be present : it is difficult to separate them :

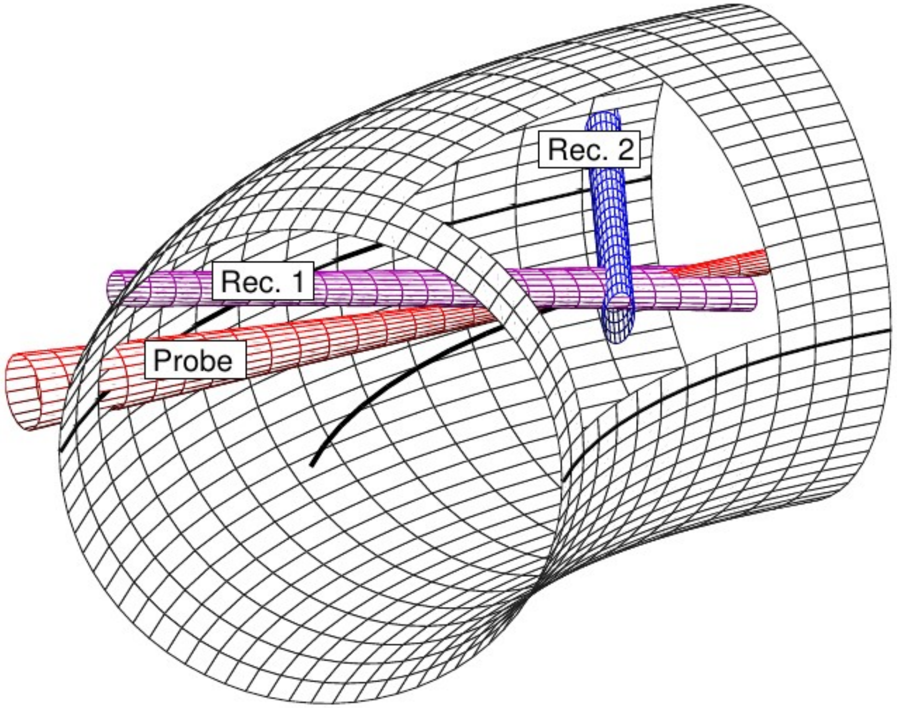
- they have to have quite different masses (heavy impurities with High Z)
- they have to have quite different temperatures (helium as fusion product)

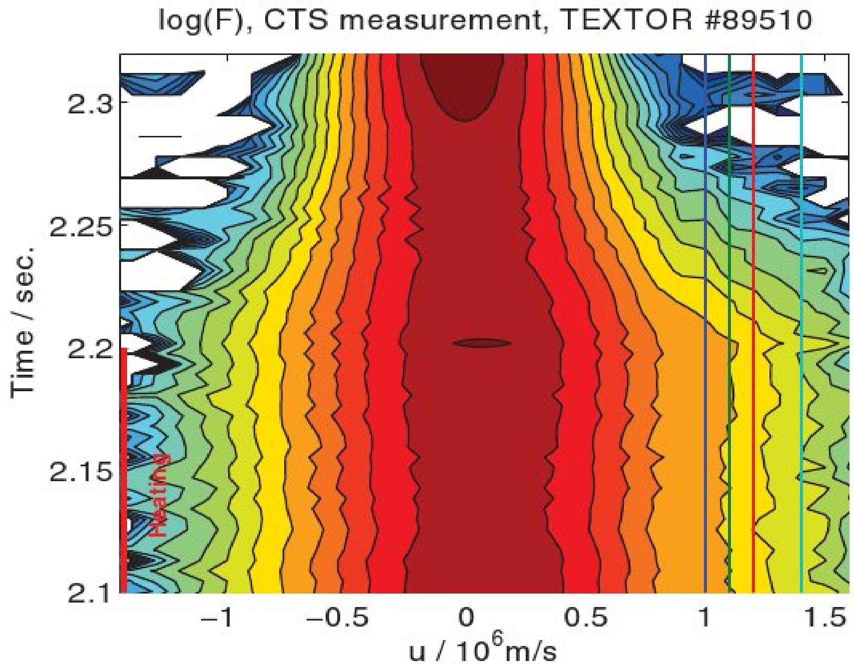
Scattering signal interpretation is not obvious because of the Salpeter approximation expression. Contrary to incoherent Thomson scattering, the scattered signal does no reproduce directly the velocity distribution.

Coherent Thomson Scattering on TEXTOR

Gyrotron 110 GHz, 150 kW
Pulsed source (4 ms ON, 4 ms OFF) to extract the signal from the ECE
Spatial resolution : 5 to 10 cm
42 spectral channels from 107 to 113 GHz
Plasma shot with heating (Neutral Beam Injection) (between t = 2.1 and t = 2.2 s)



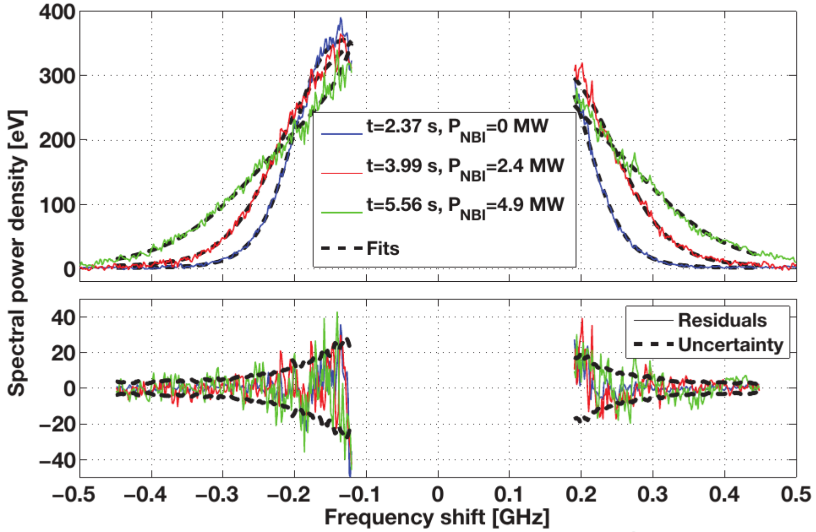


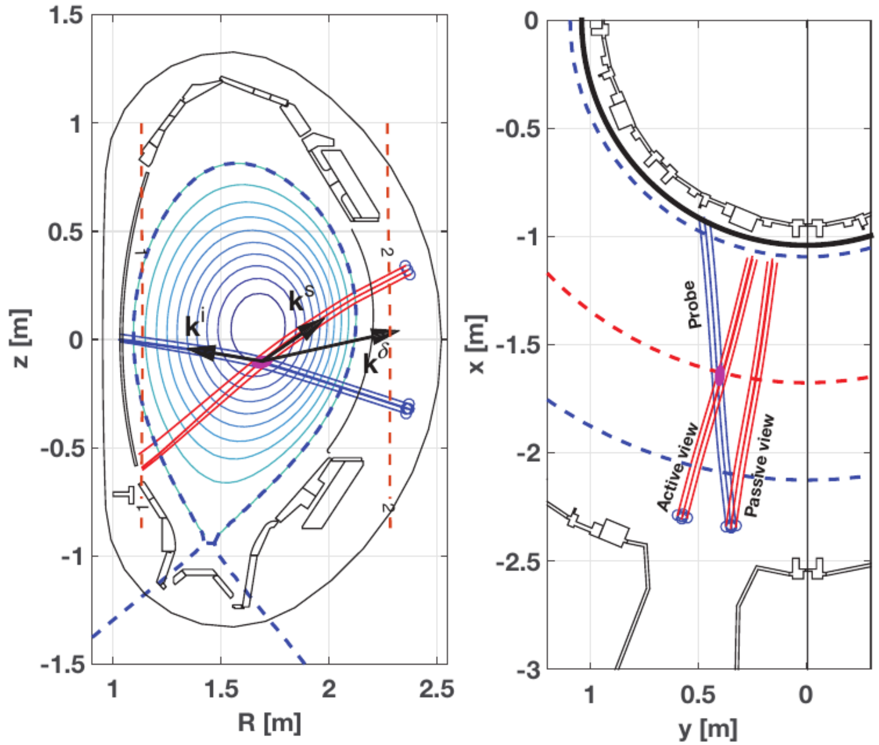


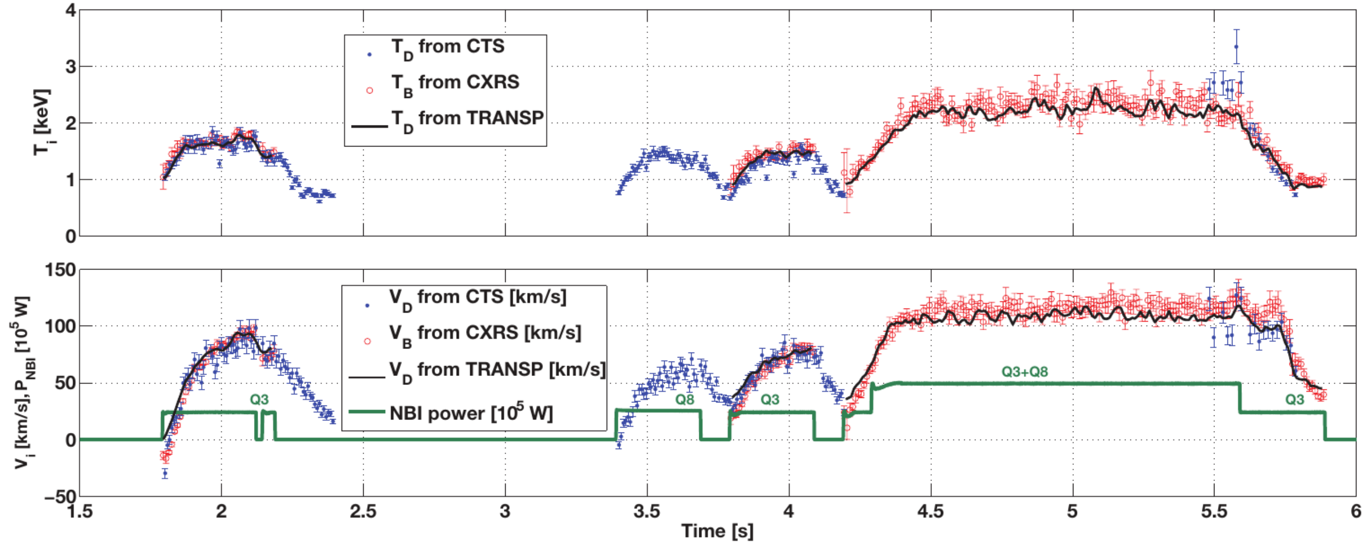
Bindslev et al., Physical Review Letters, **97**, 205005 (2006)

CTS on ASDEX-U

Source : cyclotrons used for ECRH,
105 GHz, 500 kW, 2 ms ON, 8 ms OFF

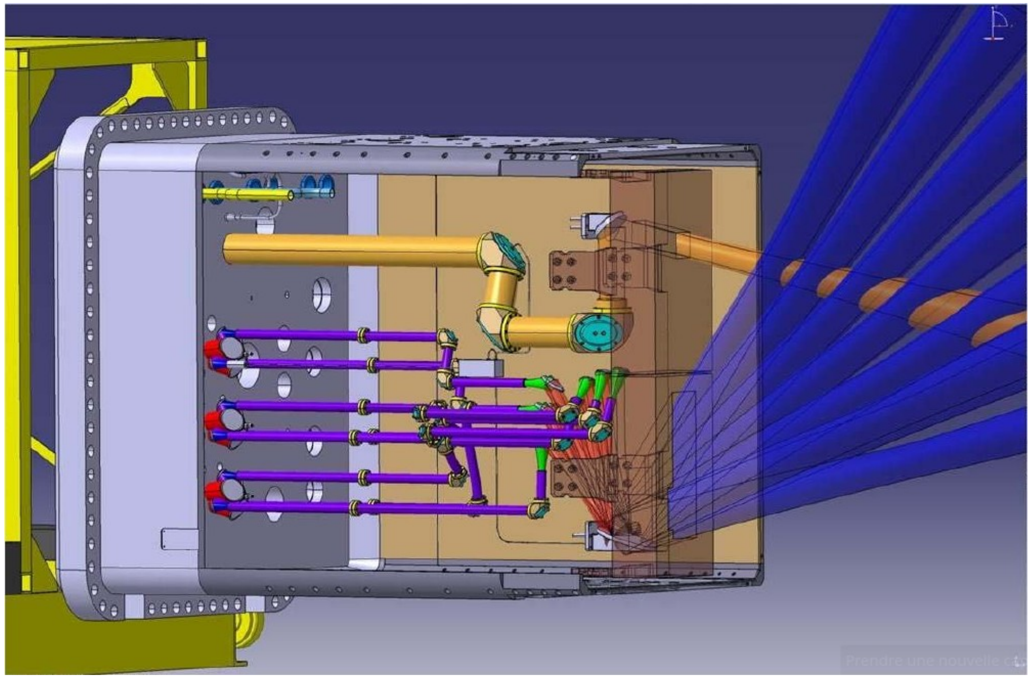


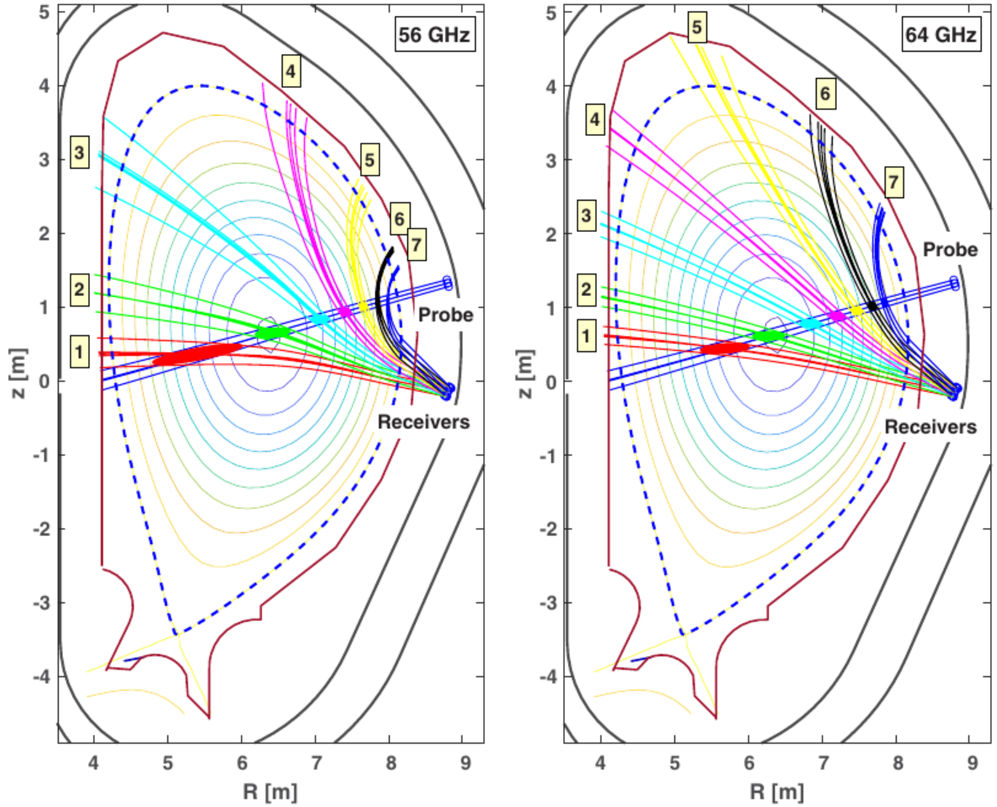




M. Stejner et al., Plasma Physics and Controlled Fusion, **57**, 062001 (2015)

ITER Coherent Thomson Scattering

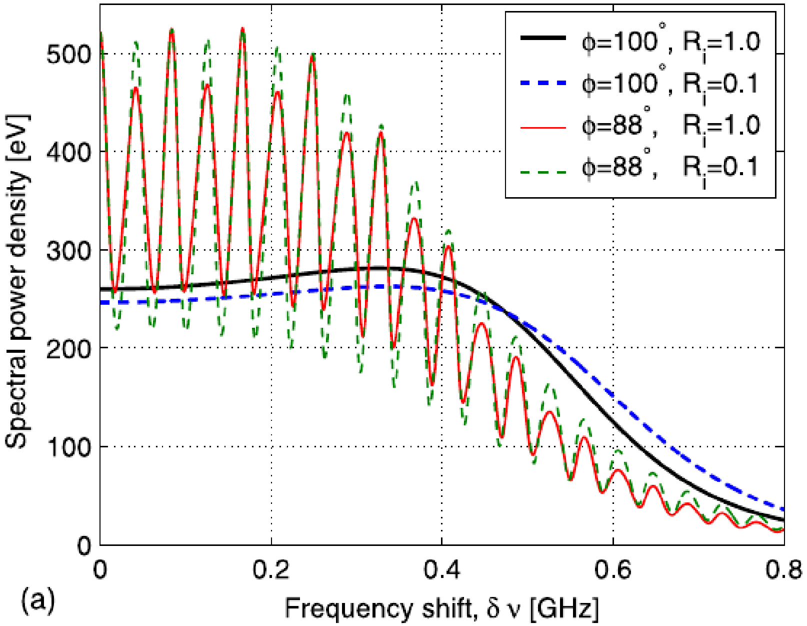


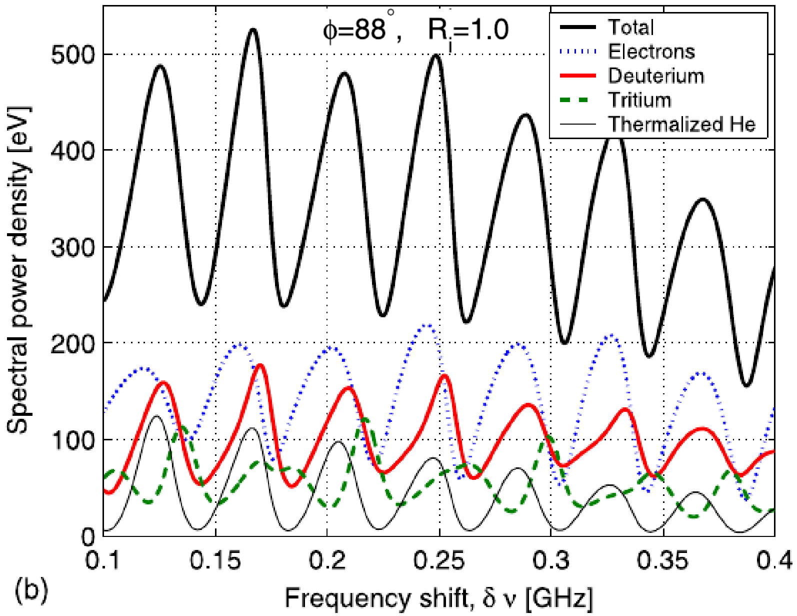


Source : cyclotrons
60 GHz, 1 MW, X mode, 0.1 s resolution

Korsholm et al., IEEE 41st Intern. Conf IRMMW-THz (2016)
Rasmussen et al., Nucl. Fusion, 59, 096031 (2019)

ITER Coherent Thomson Scattering : frequency spectra





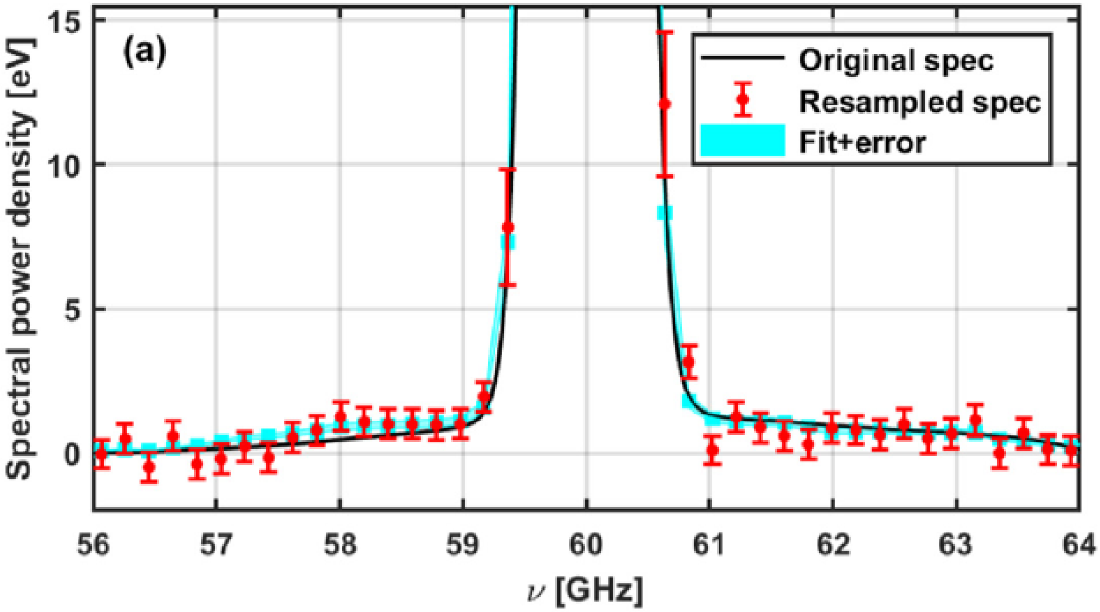
Simulated Coherent Thomson scattering spectral power density for different scattering angle and different fuel ion ratio.
Ion cyclotron frequencies intervene.
Contribution for each species depends on charge to mass ratio, thermal velocity and Larmor radius.

Description	Symbol	Unit	Value	σ_{prior}
CTS system parameters				
$\angle(\boldsymbol{k}^{\delta}, \boldsymbol{B}^{(0)})$	ϕ	$^\circ$	88	3
$\angle(\boldsymbol{k}^{i}, \boldsymbol{k}^{s})$	θ	$^\circ$	140	3
Frequency of incident radiation	ν^{i}	GHz	60	
Mode of incident radiation			X	
Mode of scattered radiation			X	
Probe source power	P_{in}	W	10^6	
Spectral power density of receiver background	P_b	eV	100	
Time resolution	T	s	0.1	
Frequency resolution	W	MHz	5	
Main plasma parameters				
Magnetic field strength	B	T	5.3	10%
Electron density	n_e	10^{19} m^{-3}	11	5%
Electron temperature	T_e	keV	26.96	10%
Ion temperature	T_i	keV	23.49	10%
Rotation velocity	V_i	m s^{-1}	8.9×10^4	30%
Plasma composition				
Fuel ion ratio	R_i		1	10^6
Fast alpha particles	n_α/n_e		0.008	10%
Thermalized alpha particles	n_{He}/n_e		0.065	10%
Hydrogen	n_H/n_e		0.01	10%
Beryllium	n_{Be}/n_e		0.005	10%
Carbon	n_C/n_e		0.005	10%
Neon	n_N/n_e		2×10^{-4}	10%
Copper	n_{Cu}/n_e		10^{-4}	10%
Krypton	n_{Kr}/n_e		10^{-4}	10%
Tungsten	n_W/n_e		10^{-5}	10%

Stejner et al., Nucl. Fusion, 52, 023011 (2012)

ITER Coherent Thomson Scattering : fast ions

Synthetic ITER CTS spectrum



Fast ion contribution :
- Alpha particles
- Neutral Beam Injection particles

The spectrum is synthesized using ray-tracing.
3 different kinds of noise is incorporated.

Uncertainties on Bayesian priors

Fuel ion ratio $R_i = n_T/(n_D + n_T)$	24%[c]
Ion temperature T_i	10%
Ion drift velocity v_i	20%
Hydrogen density n_H	14%[c]
Thermal He densities n_{He3}, n_{He4}	20%[d]
Other impurity densities (Be, Ne, W)	10%
Electron density n_e	5%
Electron temperature T_e	1%

Rasmussen et al., Nucl. Fusion, 59, 096031 (2019)

Physics and Diagnostics in Tokamak plasmas

Thomson scattering response depends on $\vec{k}$ and ω

$k\lambda_D > 1$: Incoherent Thomson Scattering

The scattering signal spectrum reproduces electron velocity distribution

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k} f_{e v_k}(-\omega/k)$$

$k\lambda_D < 1 \quad \omega_D \sim k v_{Ti}$: Coherent (or collective) Thomson Scattering

The signal spectrum is an image of the ion velocity distribution

$$S_{\vec{k}}(\omega) = \frac{2\pi}{k}\left[\left|1 - \frac{\chi_e}{1+\chi_e+\chi_i}\right|^2 f_{v_k e}\left(\frac{-\omega}{k}\right) + \sum_i \left|\frac{-\chi_e}{1+\chi_e+\chi_i}\right|^2 Z_i^2 f_{v_k i}\left(\frac{-\omega}{k}\right)\right]$$

$k\lambda_D < 1 \quad \omega_D \sim k c$: Collective Thomson Scattering

the signal spectrum might correspond plasma instabilities

with electron density fluctuations (k, kc)

$$S_{\vec{k}}(\omega) = \frac{1}{T N_s}\left|n_{eT}(\vec{k}, \omega)\right|^2 \qquad n_{\vec{k}}(t) = \iiint_{Vs} n_e(\vec{r}, t)\, e^{-i\vec{k}.\vec{r}}\, d^3\vec{r}$$

Incoherent Thomson Scattering

- Non perturbative measurements
- Direct measurement
- Spatially localized measurements with a good time resolution

Bibliography

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Chap. 10 : Diagnostics

I. H. Hutchinson, Principles of Plasma Diagnostics, 2nd ed, Cambridge Univ. Press (2002)

Chap. 7 : Scattering of electromagnetic radiation

D.H. Froula et al., Plasma Scattering of Electromagnetic Radiation, Academic Press, (1975)

J. D. Jackson, Classical Electrodynamics, J. Wiley (1975)

Chap. 9 : Simple Radiation Systems, Scattering, and Diffraction

Physical constants

$k_B = 1,38\ 10^{-23} JK^{-1}$: Boltzmann constant
$h = 6.62\ 10^{-34} Js$: Planck constant
$C = 2,99\ 10^{8} ms^{-1}$: speed of light in vacuum
$\varepsilon_0 = 8.85\ 10^{-12} Fm^{-1}$: vacuum permittivity
$\mu_0 = 4\pi\ 10^{-7} Hm^{-1}$: vacuum permeability
$q_e = 1,60\ 10^{-19} C$: elementary charge
$m_e = 9,11\ 10^{-31} kg$: electron mass
$r_e = \frac{1}{4\pi\epsilon_0}\frac{q_e^2}{m_e C^2} = 2.82\ 10^{-15} m$: electron classical radius
$N_A = 6,022\ 10^{23} mol^{-1}$: Avogadro constant
$m_u = 1,66\ 10^{-27} kg$: atomic mass unit

- Standard parameters

$T_0 = 273.15 K$: standard air temperature ($0°C$)
$P_0 = 1,013\ 10^{5} Pa$: standard air pressure
$n_0 = 2,69\ 10^{25} m^{-3}$: ideal gas molecular density at T_0 and P_0

- Units

$1\, Torr = \frac{1,013\ 10^5}{760} Pa = 133,3\, Pa$: pressure corresponding 1 mm of mercury
$1\, eV = \frac{1,6\ 10^{-19}}{1,38\ 10^{-23}} K = 1,16\ 10^{4} K$