INTRODUCTION TO LANGMUIR PROBES IN MAGNETIZED PLASMAS

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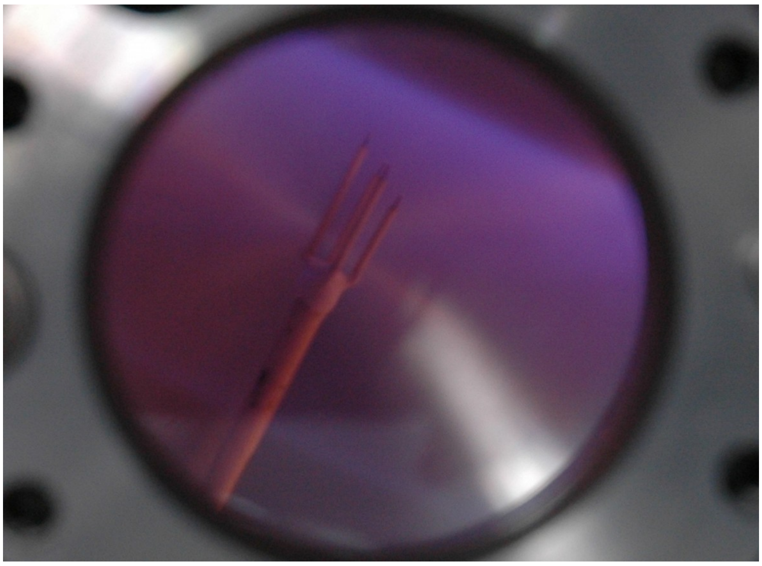

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Introduction

A Langmuir probe is a conductor that is immersed in the plasma. We measure the current at the probe output as a function of the probe polarization potential with respect to the plasma potential. From this probe current-potential characteristic we can deduce some plasma parameters around the probe. A probe plasma interaction model is necessary to evaluate the plasma characteristics from the measurements.

Probe Measurements are intrusive: they interact with the plasma. We focus on cold, weakly collisional plasmas. We present the effects of the magnetic field on the probe measurement.

The first part exposes an basic model of the probe.

The second part describes the implementation of the probe measurement.

The third part exposes a probe sheath model. This model is necessary to estimate the ion and electron flows at the probe.

Contents

1 Elementary probe model

We first expose an elementary model of the probe immersed in the plasma in order to describe the general behavior. Some results are given with no model. They are described in part 3. This part exposes the interaction between the plasma and the probe.

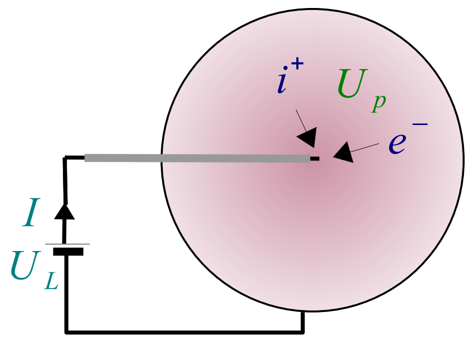
In order to analyze the probe current-voltage characteristic, it is necessary to determine the ion and electron flows to the probe.

1.1 Particle flow to the probe

We consider a cold plasma, consisting of single charged positive ions i^+ and electrons e^- . Interaction with neutrals are neglected.

The ion temperature is very low compared to the electron one : $T_i \ll T_e$.

The probe surface is considered completely absorbing for the charged particles that reach it (the ions are recombined on the surface).



Since the Langmuir probe potential U_L is different from the plasma potential U_p , the probe has a different effect on the plasma electrons and ions. Because of the potential difference, over a certain distance around the probe, the quasi-neutrality of the plasma is no longer verified: it is the sheath that surrounds the probe. If the probe potential is lower than that of the plasma, the sheath mainly consists of ions. Electrons dominate in the opposite case.

The particle flow reaching the probe depends on the probe shape, the sheath thickness, the direction of the possible magnetic field.

If the probe is electrically connected to the same ground as the plasma, the difference between the electron and ion flows reaching the probe generate an electric current I at the probe output.

The probe current-voltage characteristic helps to determine the ion and electron densities, the electron temperature and the plasma potential in the vicinity of the probe.

1.a Ion sheath

We are in the case the probe potential U_L is lower than the plasma potential U_p .

The ion current at the entrance to the sheath surrounding the probe is called the ion saturation current I_{iS} .

Ion saturation current

This current is the product of the elementary charge q_e by the ion flow.

The ion flow is the product of the probe collection area A_i , by the ion flux. The probe ion collection surface is the surface at the sheath boundary with the surrounding plasma.

The ion flux is the product of the ion density at the sheath surface, n_{is} , by the mean ion velocity at the entrance to the sheath v_{is} .

By convention, the current is positive for the electron part :

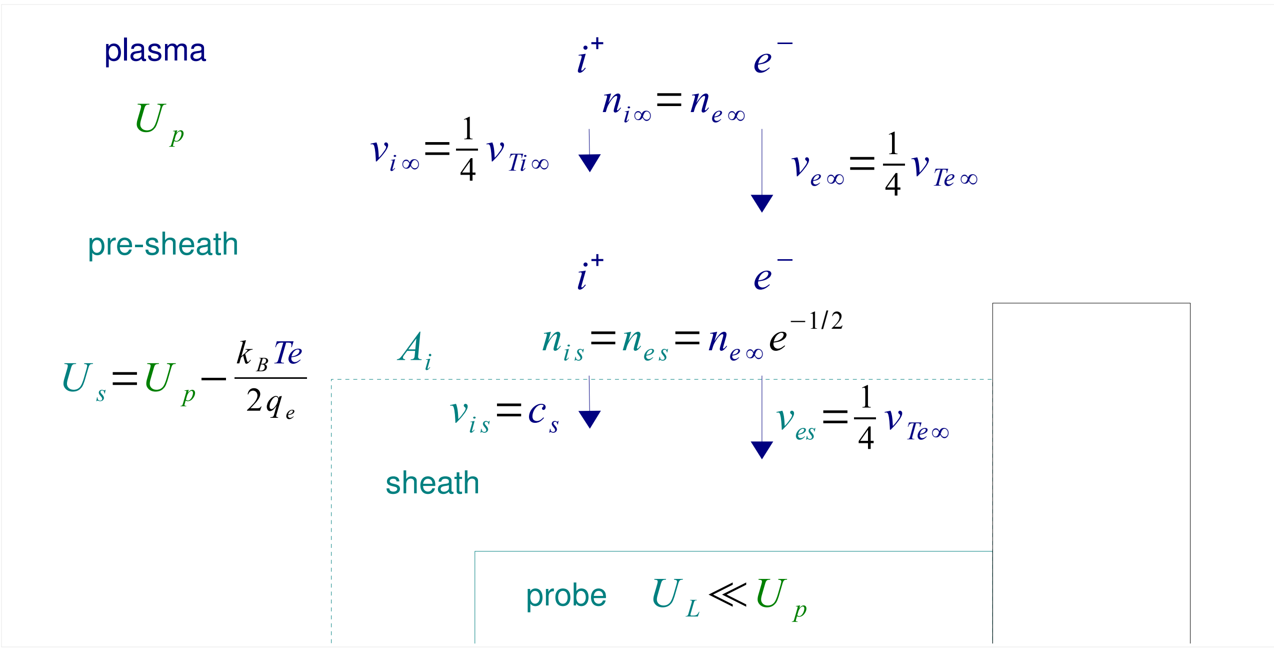
$$I_{iS}=-q_e n_{is} v_{is} A_i \quad .$$

Bohm criterion on the ion sheath surface

The detailed sheath model in part 3 shows that the ions can not enter the sheath with a velocity smaller than the Bohm velocity (i.e. the ion acoustic velocity) :

$$v_{is}=c_s=\sqrt{\frac{k_B Te}{m_i}} \quad .$$

This is the Bohm criterion.



The plasma ion thermal velocity is lower than this velocity (because their mass is much larger and their temperature is lower than the electron ones). Around the sheath, in a zone called pre-sheath, they undergo a first electric acceleration towards the sheath. The acceleration is provided by a potential difference between the plasma core U_p and the sheath surface :

$$U_s=U_p-\frac{k_B Te}{2q_e}$$

Due to this acceleration, and because of the mass conservation, the ion density at the surface of the sheath is lower than that in the quasi-neutral plasma far from the probe, beyond the pre-sheath:

$$n_{is}=n_{e\infty}e^{-1/2}$$

The correction factor is established in the detailed sheath model.

The ion saturation current is :

$$I_{iS} = -q_e n_{e\infty} e^{-1/2} A_i \sqrt{\frac{k_B Te}{m_i}}$$

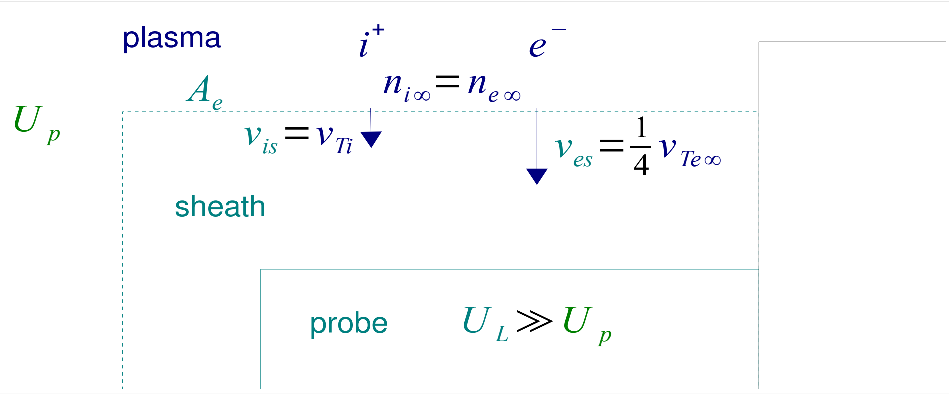
The sheath model shows the ion sheath collection area, A_i , depends on the probe geometry, the probe potential and, if applicable, the magnetic field.

The ion sheath thickness e_{is} depends on the potential difference between the probe and the plasma $U_\Delta = U_L - U_p$ and the Debye length λ_{De} :

$$e_{is} = 1.02 \left[\frac{q_e |U_\Delta|}{k_B T_e} \right]^{3/4} \lambda_{De}$$

1.b Electron sheath

The probe potential U_L is larger than the plasma potential U_p , we are in the case of the electron sheath. Ions are repelled from the probe.



Electron saturation current

The electron saturation current is calculated by the same product, estimated at the plasma electron sheath surface:

$$I_{eS} = q_e n_{es} v_{es} A_e \quad .$$

The electron thermal velocity is large enough at the sheath boundary : no pre-sheath is necessary. The electron parameters at the sheath boundary are the same as in the surrounding plasma :

$$n_{es} = n_{e\infty}$$

The mean velocity component towards the probe is a fourth of the mean thermal velocity modulus:

$$v_{es} = \frac{1}{4} v_{Te\infty} = \frac{1}{4} \sqrt{\frac{8 k_B T_e}{\pi m_e}} \quad .$$

The electron collection area A_e may differ from the ion collection area depending on the approximations.

$$I_{eS} = \frac{1}{4} q_e n_{e\infty} A_e \sqrt{\frac{8 k_B T_e}{\pi m_e}} \quad .$$

The electron sheath thickness e_{es} depends on the potential difference between the probe and the plasma $U_\Delta = U_L - U_p$ and the Debye length λ_{De} :

$$e_{es}(U_\Delta)=1.26\left[\frac{q_e U_\Delta}{k_B T_e}\right]^{3/4}\lambda_{De}$$

Electron ans ion saturation currents comparison

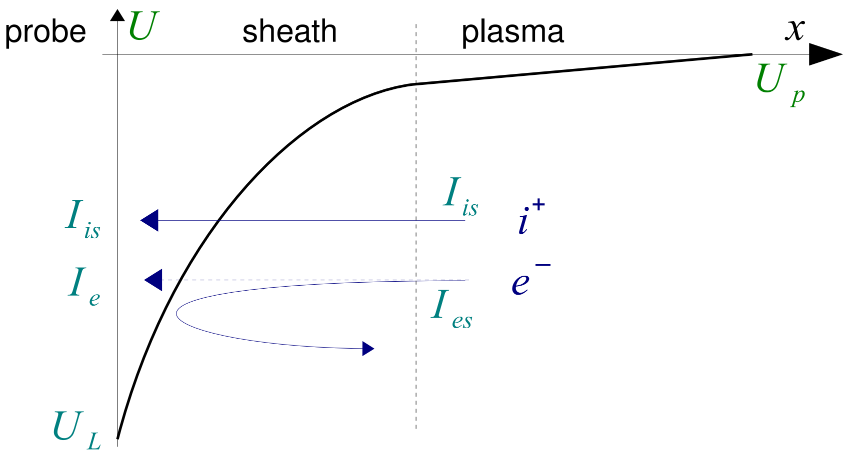
For a non magnetized plasma, the ion and electron collection areas are of the same order of magnitude (less verified in magnetized plasma). Since the constant factors are also of the same order, the main ratio between the expressions of the ion and electron saturation currents is the inverse of the square root of the ion to electron mass ratio $\sqrt{m_i/m_e}$. The electron saturation current is, with some exceptions, much more intense than the ion current.

1.2 Probe current-voltage characteristic

The ion and electron flows reaching the probe depend on the difference between the probe potential, U_L , and the plasma potential around, U_p .

2.a $U_L \le U_p$ Probe potential lower than plasma potential

We first study the case where the potential of the probe U_L is lower than the plasma potential U_p : $U_L \le U_p$.



The ions are attracted by the probe: all the ions at the entrance of the sheath reach the probe, the ion probe current I_i is the ion saturation current I_{is} :

$$I_i(U_L)=I_{iS}(U_L)\ .$$

On the other hand, part of the electrons entering the sheath are repelled in the sheath depending on their kinetic energy.

The electrons have a Maxwellian energy distribution in the plasma :

$$f_e(E)=\frac{n_{e\infty}}{k_B T_e} e^{\frac{-E}{k_B T_e}}\ .$$

All electrons whose kinetic energy E is smaller than the electrical potential energy across the sheath $q_e(U_p - U_L)$ are repelled. The remaining density part, that which reach the probe, is:

$$n_e = n_{e\infty} e^{\frac{-q_e}{k_B T_e}(U_p - U_L)}\ .$$

The electron current part I_e reaching the probe is the electron saturation current I_{es} reduced in the same proportions :

$$I_e(U_L)=I_{eS}(U_L)e^{\frac{-q_e}{k_B T_e}(U_p-U_L)} .$$

The total current I is the sum of the ion and electron currents.

$$I(U_L)=I_{iS}(U_L)+I_{eS}(U_L)e^{\frac{-q_e}{k_B T_e}(U_p-U_L)}$$

2.b $U_L > U_p$ Probe potential larger than plasma potential

We consider the opposite case where the probe potential is larger than the plasma potential : $U_L > U_p$.

In this case, the ions are repelled by the sheath. Since the ion temperature is very low, all ions are repelled :

$$I_i(U_L)=0 .$$

All the electrons reach the probe :

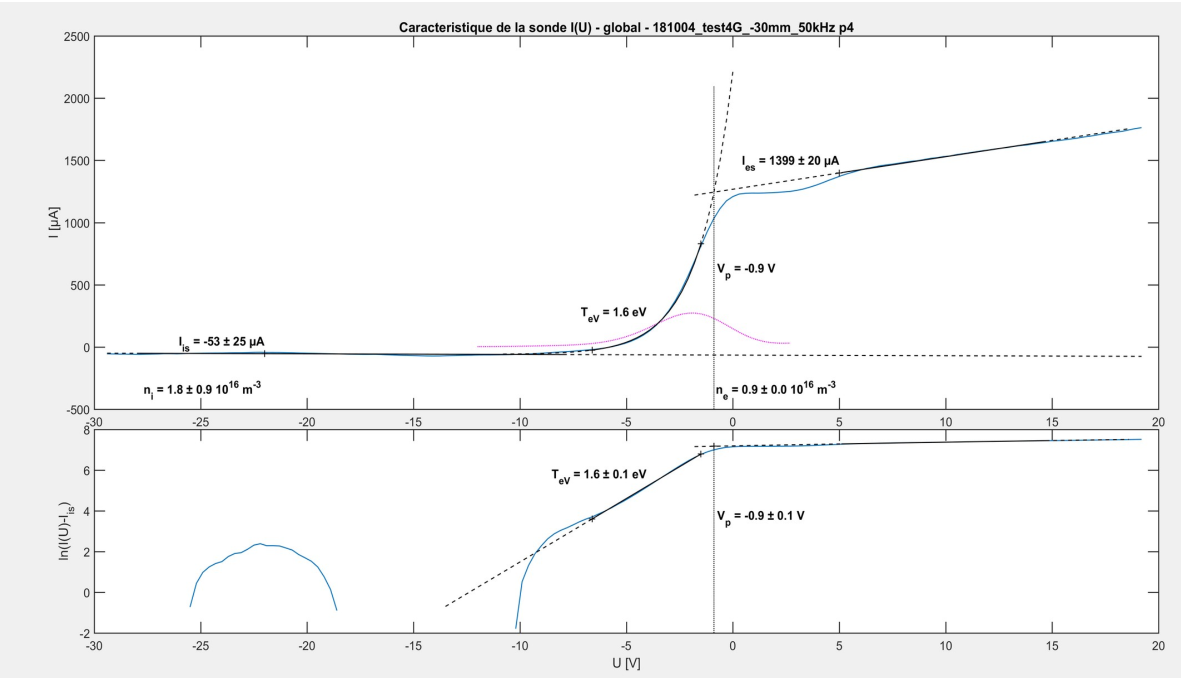
$$I_e(U_L)=I_{eS}(U_L) .$$

The current at the probe output corresponds to the electron saturation current :

$$I(U_L)=I_{eS}(U_L) .$$

2.c The current-voltage characteristic shape

The graphs below show a typical probe current-voltage characteristic.



The first graph represents the current as a function of the polarization voltage : $I(U_L)$.

The second one represents the logarithm of the current from which the ionic saturation current has been subtracted (estimated from the data) to keep only the electron part of the current:

$$f(U_L)=\log(I(U_L)-I_{iS}(U_L)) .$$

In the intermediate zone, below the plasma potential, where the electron current is much larger than the ion saturation current, this function has the form:

$$f(U_L)=\log(I_{es}(U_L))-\frac{q_e}{k_B T_e}(U_p-U_L) \quad .$$

This function has an affine form (the $I_{es}(U_L)$ logarithm variation is neglected). the function slope is inversely proportional to the electron temperature T_e :

$$\frac{df}{dU}(U_L)=\frac{q_e}{k_B T_e} \quad .$$

The function $f(U_L)$ reaches the electron saturation current value when the voltage is equal to the plasma potential : $f(U_L)=\log(I_{es}(U_L))$.

Above the plasma potential, the function keeps the electron saturation current value. This value still varies slowly with the probe potential because the probe collection area depends on the probe to plasma potential difference. It is possible to deduce the potential of the plasma because of this property.

Derivation of plasma parameters

We can deduce from the probe current-voltage characteristic the main plasma parameters around the probe.

The ion and electron saturation currents are deduced from the current values corresponding respectively to the lowest and highest potentials.

In the intermediate range, the characteristic should show an exponential shape. In this case, the electron temperature is deduced from the exponential rate parameter. The plasma potential corresponds to the transition from the exponential part to the electron saturation part.

Knowing the electron temperature, it is possible to estimate the electron thermal velocity and the Bohm velocity. From these velocities we can deduce the ion and electron densities from the saturation currents, provided that the probe ion and electron collection areas can be estimated.

1.3 Magnetic Field Effects

The magnetic field acts on the Langmuir probe measurements through the estimation of the the probe ion and electron sheath collection surface.

Since the ions and electrons are confined by the cyclotron motion around the guiding center trajectory along the magnetic field lines, the probe particle collection area is limited by the shadow of the probe across the direction of the magnetic field. The collection surface also includes the sheath that surrounds the probe.

To this surface, in certain cases, we add a thickness equal to the particle mean Larmor radius. This extension depends on the probe geometry.

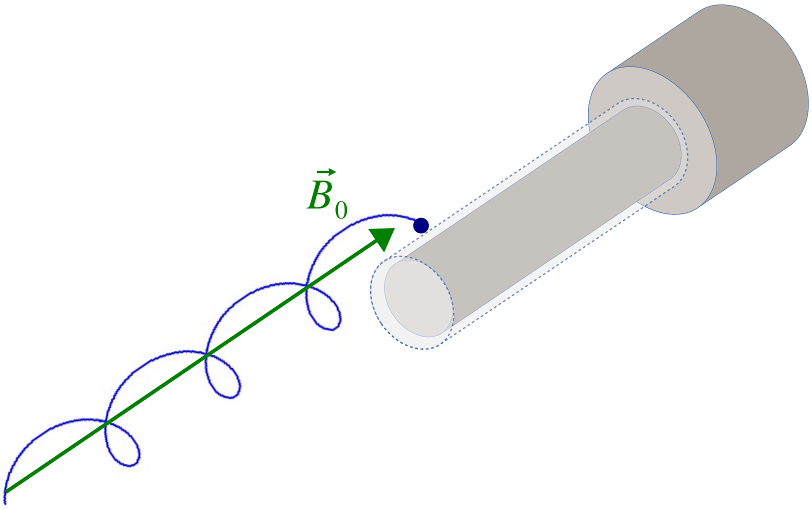
3.a Cylindrical probe along the magnetic field axis

We consider a cylindrical probe whose axis is along the magnetic field $\vec{B}_0$.

The probe radius is ρ_L and length, l_L . The probe interacts with the plasma on one side of the cylinder. The other side is usually used for the probe support and external connection.

The collection area depends on the particle species, ion or electron, on the potential of the probe and on the magnetic field.

Probe ion collection area



For the ion collection, the collection area includes the probe surface:

$$A_L = \pi \rho_L^2$$

It is necessary to add to this area, the probe sheath thickness $e_{is}(U_L)$:

$$A_{si} = \pi \left[\rho_L + e_{is}(U_L)\right]^2 \ .$$

The ion Larmor radius increases the probe collection area.

The ion cyclotron angular frequency $\omega_{ci} = 2\pi f_{ci}$ is : $\omega_{ci} = \frac{q_e B_0}{m_i}$

The ion temperature, even if it is small, induces a significant ionic Larmor radius, compared to the probe sheath size.

The mean thermal velocity along each direction v_{Ti} is :

$$v_{Ti} = \sqrt{\frac{3 k_B T_i}{m_i}} \ .$$

The ion mean Larmor radius ρ_{cTi} is : $\rho_{cTi} = \frac{v_{Ti}}{\omega_{ci}}$.

The mean particle path per cyclotron gyration l_{cTi} along the magnetic field line is :

$$l_{cTi} = \frac{v_{Ti}}{\nu_{ci}} = 2\pi \rho_{cTi} \ .$$

We consider the probe is longer than this path, $l_L > l_{cTi}$.

All ions whose guiding center is closer to the probe sheath than the mean Larmor radius ρ_{ci} intercept the sheath. In this case, the ion collection air is :

$$A_i = A_{si} = \pi \left[\rho_L + e_{is}(U_L) + \rho_{cTi}\right]^2 \ .$$

The probe ion collection area depends on the probe potential and the magnetic field magnitude.

Probe electron collection area

For the electron collection, the electron Larmor radius is much smaller than the ion Larmor radius. It is generally negligible compared to the size of the probe and sheath.

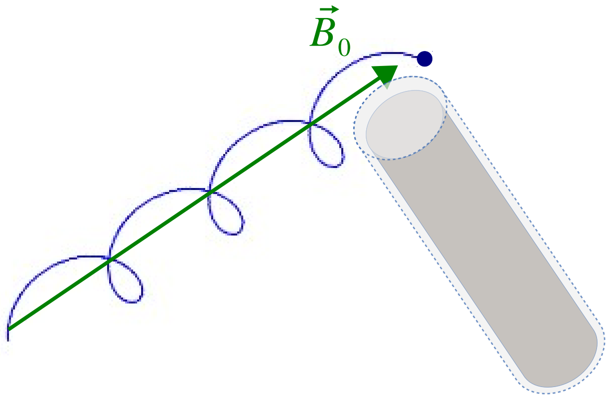
The probe electron collection area is limited to the surface of the electron sheath around the probe:

$$A_e = A_{se} = \pi\left[\rho_L + e_{es}(U_L)\right]^2 \ .$$

3.b Cylindrical probe perpendicular to the axis of the magnetic field

If the cylindrical probe is perpendicular to the magnetic field lines, the probe collection areas are very different.

Ion collection area



The probe surface perpendicular is a rectangle of length l_L and width $2\rho_L$. The probe can be reached from both sides. The probe collection area in the direction of the field is for this configuration:

$$A_p = 4\rho_L l_L \ .$$

The probe area including the sheath (the sheath is only present on one side in the length direction) :

$$A_{si} = 4\left(\rho_L + e_{is}(U_L)\right)\left(l_L + e_{is}(U_L)\right) \ .$$

The finite ion mean Larmor radius also applies:

$$A_{\rho i} = 4\left(\rho_L + e_{is}(U_L) + \rho_{cTi}\right)\left(l_L + e_{is}(U_L) + \rho_{cTi}\right) \ .$$

If the probe size along the magnetic field line is longer than the ion guiding center mean path per cyclotron gyration, $2\left(\rho_L + e_{is}(U_\Delta)\right) > l_{cTi}$, the collection area also includes the thickness ρ_{cTi} around the sheath :

$$A_i = A_{\rho i} \ .$$

In the opposite case $2\left(\rho_L + e_{is}(U_\Delta)\right) < l_{cTi}$, the ion portion intercepting the sheath depends on the ratio between the probe size along the magnetic field line and the ion guiding center mean path per cyclotron gyration:

$$p = \frac{2\left(\rho_L + e_{is}(U_\Delta)\right)}{l_{cTi}} \ .$$

The collection area is then :

$$A_i = A_{si} + min\left[\frac{2\left(\rho_L + e_{is}(U_\Delta)\right)}{l_{cTi}}, 1\right]\left(A_{\rho i} - A_{si}\right) \ .$$

Electron collection area

We neglect the electronic Larmor radius.

The probe collection area including the sheath :

$$A_e = 4\left(\rho_L + e_{es}(U_\Delta)\right)\left(l_L + e_{es}(U_L)\right) \ .$$

1.4 Floating potential

If the probe is not connected to the same ground as the plasma, no current comes out of the probe. In this case, the probe is electrically charged so its own potential guarantees the absence of net current to it: this is the probe floating potential U_f :

$$I(U_f)=0 \ .$$

The floating potential U_f is different from the plasma potential U_p . When the probe is at plasma potential, no particle is repelled. Since the electron velocity is much larger than the ion one, as the ion and electron densities are close, the electron flow to the probe is much larger than the ion flow. The probe becomes negatively charged with electrons. Its potential decreases. The floating potential is therefore lower than the plasma potential $U_f<U_p$.

It is fixed by the zero current condition :

$$I_{iS}(U_f)+I_{eS}e^{\frac{q_e}{k_B T_e}(U_f-U_p)}=0 \ .$$

We use the expressions for the ion and electron saturation currents :

$$\frac{1}{4}q_e n_{e\infty}A_e(U_f)\sqrt{\frac{8k_B T_e}{\pi m_e}}e^{\frac{q_e}{k_B T_e}(U_f-U_p)}=q_e n_{e\infty}e^{-1/2}A_i(U_f)\sqrt{\frac{k_B Te}{m_i}} \ .$$

We deduce the floating potential :

$$U_f=U_p-\frac{k_B T_e}{2q_e}\left(\frac{1}{2}\ln\frac{m_i}{2\pi m_e}+\ln\frac{A_e(U_f)}{A_i(U_f)}\right) \ .$$

The floating potential U_f differs from the plasma potential U_p by several times the factor $k_B T_e/2q_e$ (half of the electron temperature expressed in eV).

Its knowledge alone does not allow to know precisely the plasma potential. In non magnetized plasma, the ion and electron collection areas are close: the term inside the logarithm essentially depends on the particle mass ratio. In magnetized plasma, the finite Larmor radius effect on the ion collection area must be taken into account.

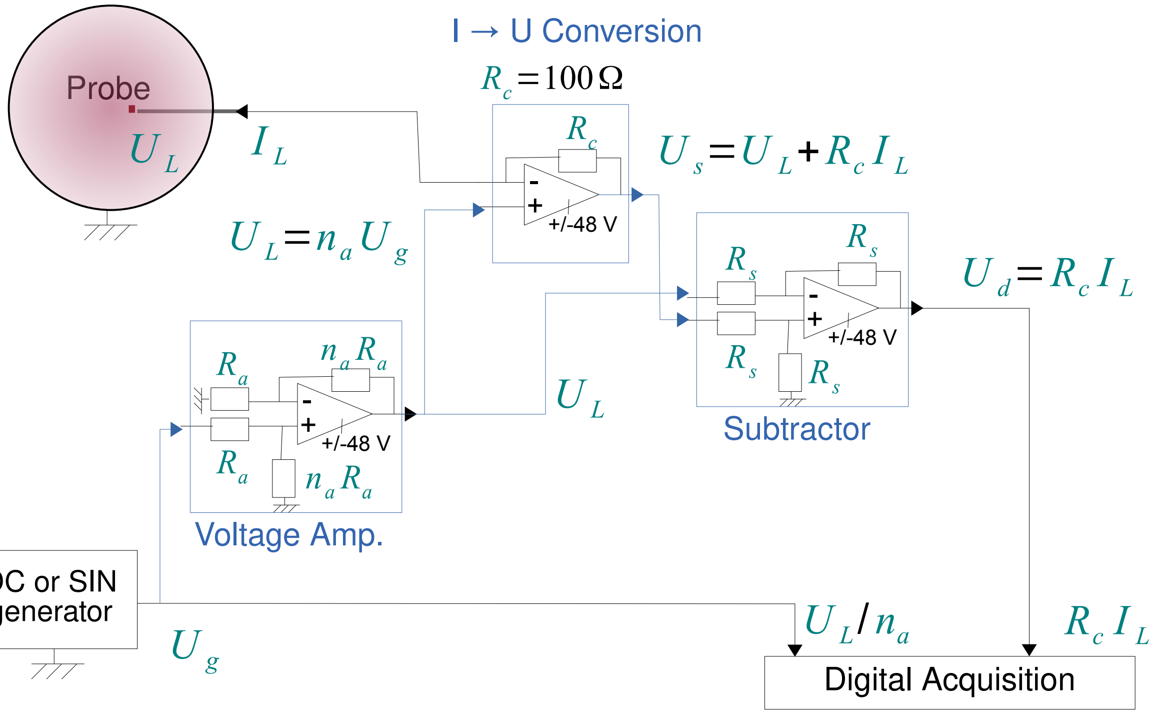
2 Probe Measurement Practice

The probe electronic assembly and the data analysis for mean and fluctuation measurements are described.

2.1 Probe polarization and current measurement

The Langmuir probe must be polarized over a wide voltage range, between $U_{Lmin}=-30\,V$ and $U_{Lmax}=20\,V$ in order to reach the saturation current zones over a fairly wide voltage range.

In order to simultaneously impose a probe polarization voltage on the probe and read the current coming from the probe, an electronic assembly based on operational amplifiers is used.



- The main component of this assembly is a <u>current-voltage converter</u>, allowing the probe polarization. The conversion resistor R_c choice (here $R_c=100\,\Omega$) depends on the maximum expected current, in order to guarantee a maximum output voltage of a few volts.
- Upstream, a <u>voltage amplifier</u> (factor n_a , $n_a=10$, input resistance R_a , feedback resistance n_aR_a) allows the use of a standard analog signal generator (voltage range $U_g\in[-3\,V,2\,V]$). It is also easier to acquire such generator voltage U_g in synchronization with the probe current measurement. The voltage amplifier output voltage is $U_L=n_aU_g$.
- At the output of the current-voltage converter, the voltage is $U_s=U_L+R_cI_L$. A

voltage subtractor (common resistance R_s) is used to subtract the probe polarization voltage : the converted probe current only remains. A digital acquisition card is used to simultaneously record the generator voltage, $U_g = U_L / n_a$, and the voltage resulting from the probe current conversion $U_d = R_c I_L$.

A point-by-point polarization voltage and time mean current measurements give the probe mean current-voltage characteristic. The characteristic analysis gives the time mean parameters of the plasma.

2.2 Dynamic Probe Current Measurements

With the Langmuir probe, it is also possible to observe fluctuations in plasmas.

2.a Fixed potential measurement

The easiest fluctuation measurement is to acquire the probe current at constant probe potential. If the chosen potential is always higher than the plasma potential, the current measurement corresponds to the probe electron saturation current. This current mainly varies with the electron density. It also depends on the electron temperature, because of the role of the electron velocity in the current.

2.b Dynamic characteristics

We want to perform dynamic measurements of electron density, temperature and plasma potential. The probe current-voltage characteristic measurements can be performed by very rapidly polarizing the probe over the full characteristic voltage range.

For the characteristic to be exploitable, the plasma must be considered as a frozen fluid for the time necessary to measure this characteristic. The ion and electron densities, electron temperature and plasma potential must be considered constant over this voltage sweep time.

The polarization frequency of these measurements is limited by the response time of the electronic assembly operational amplifiers. In order to avoid the presence of harmonic frequencies higher than the fundamental, a sinusoidal signal shape is used, rather than a triangular signal.

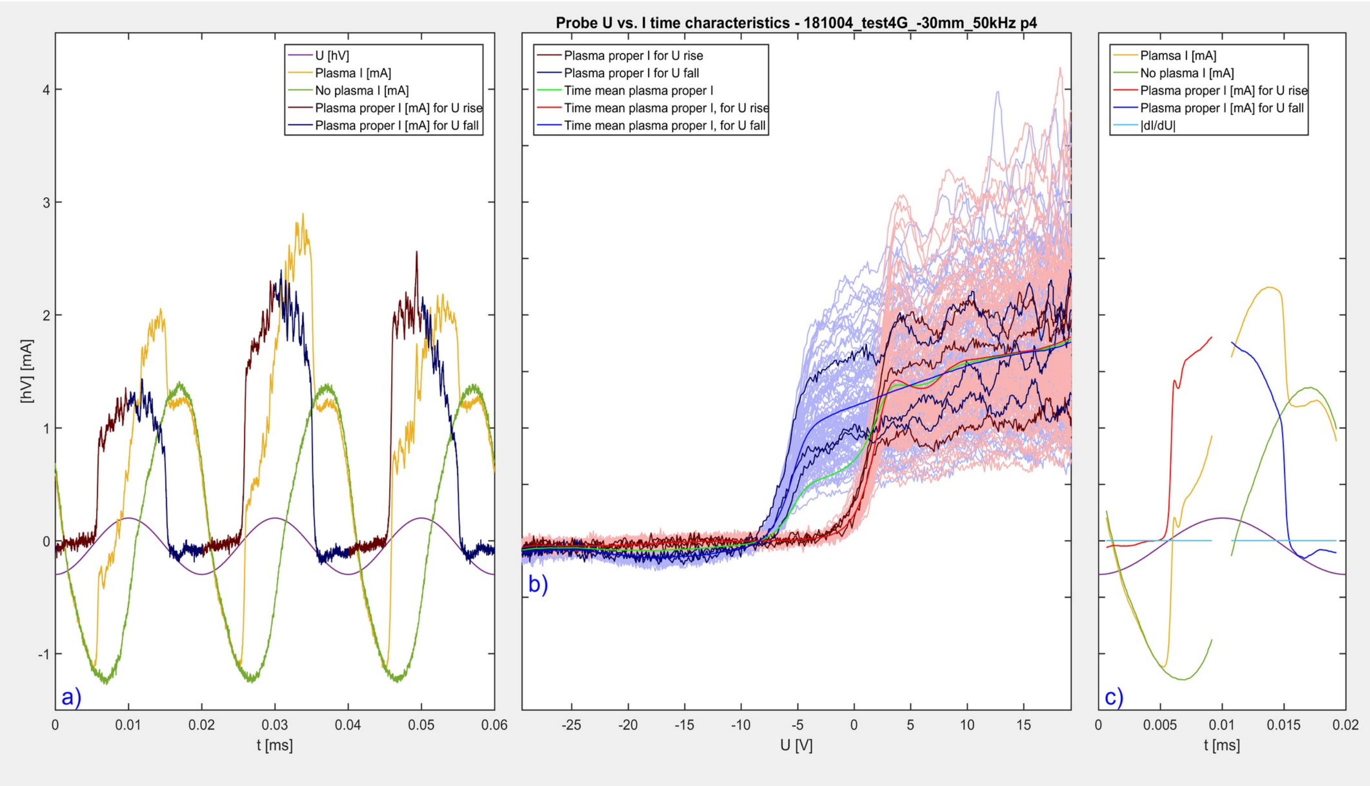
For op-amps with a rise time of $20\,V/\mu s$, the generator can be used up to the frequency of $50\,kHz$. At this frequency, the duration of a rising (or falling) voltage edge is equal to half a period of the generator signal, $10\,\mu s$. This voltage sweep duration is sufficiently short if the plasma fluid fluctuations have a maximum frequency lower than $50\,kHz$.

This dynamic measurement can also produce the time mean plasma parameters. The probe time mean characteristic is deduced from current and voltage time data : for each polarization voltage, the corresponding currents are averaged. The total duration of the measurement must then be long enough to validate the ergodic assumption. The generator frequency is not important for these averaged measurements.

In order to estimate the biases due to the high frequency diagnosis operation, we can compare the dynamics time mean measurements with the generator frequency of $50\,kHz$, with another one obtained at a much lower frequency $500\,Hz$. The low frequency measurement of the generator does not allows to observe the dynamics of the plasma.

2.3 Dynamic signal correction

The graphs below show time signals of the probe polarization and current for a generator at $f_g = 50\,kHz$.



The graph a) shows time signals during 3 generator periods. The yellow plot corresponds to the probe current (in mA). The sinusoidal signal in purple plot shows the generator signal (in hV). The current signal is very asymmetrical with respect to the generator signal. However, the current response of the plasma should mainly depend on the polarization voltage.

3.a Measurements with and without plasma

A probe current measurement without plasma (green plot) shows that the diagnosis has a significant measurement bias: this bias is mainly due to the leakage currents in the op-amps, and to the capacitive nature of the probe for the circuit (the probe current is almost in quadrature with respect to the polarization voltage). In order to compensate for these effects, the probe current measured without plasma is subtracted from the current measured with the plasma. The measurement without plasma is performed at another time with the same device. The subtraction of the 2 signals requires a correct generator signal synchronization, so that the polarization conditions are identical on both signals. The generator frequency is stable enough that synchronization is possible throughout the signal.

The current difference corresponds to the red and blue signal. The signal part in red corresponds to the intervals when the generator voltage increases, the blue part, to the complementary condition. This differentiation highlights the hysteresis on the signal.

3.b Delay of the current over voltage measurements

The graph b) shows the same signal, as a function of the probe polarization voltage, in order to highlight the current-voltage characteristics. The semi-transparent signal corresponds to the continuation of the signal in time after the 3 first generator periods.

For each polarization voltage, the currents measured over time are averaged: this is a green curve. The same mean value is evaluated, but differentiating between the rising (in red) and falling (in blue) edges of the generator: these are the lighter red and blue curves.

The graph c) shows these same time means (as well as the raw signal means), but represented again as a function of time.

For this measurement carried out with a generator high frequency ($f_g = 50\,kHz$), hysteresis appears on the current-voltage characteristic between the measurements corresponding to the time increasing probe polarizations and those corresponding to the decreasing ones. This hysteresis is very pronounced on the central part, where the current varies rapidly with the voltage. This hysteresis is partly due to a delay between the probe polarization potential and the probe current measurements. This delay is due to the round trip time of the signal from the generator to the probe and the response time of the current-voltage conversion electronics. This delay is estimated by seeking to minimize the hysteresis on the central part of the characteristic. It is estimated at a value close to $0.5\,\mu s$. This delay correction shows very small variations with the experiments ($\pm 20\%$). For low frequency measurements ($f_g = 500\,Hz$), this delay has no significant effect.

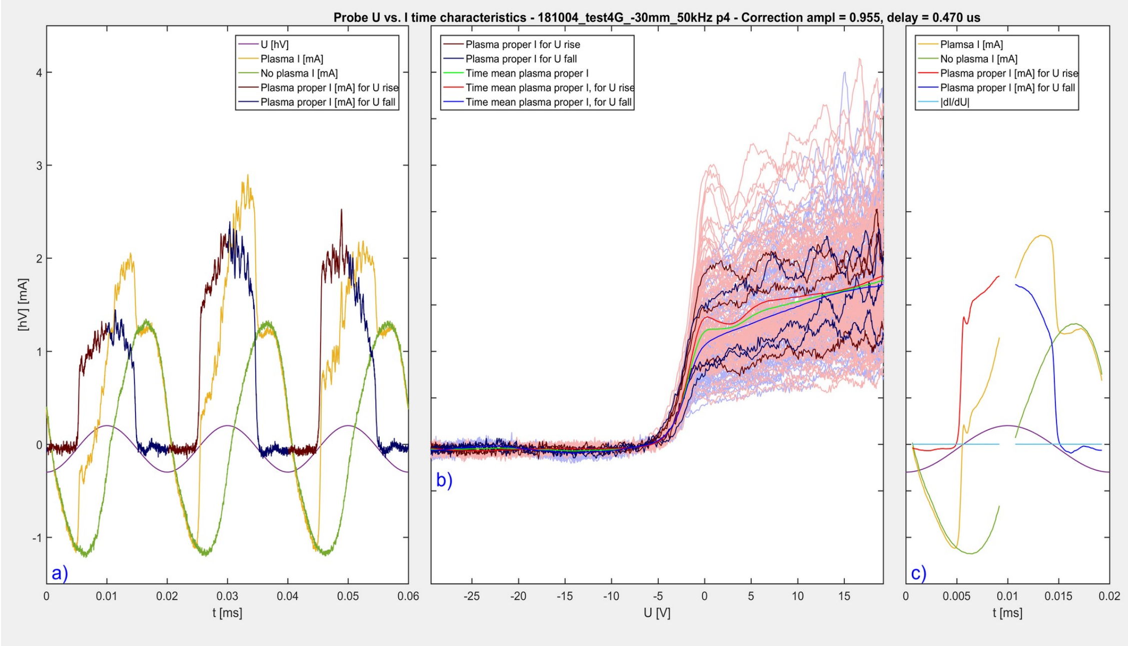
3.c Optimization of current amplitude without plasma

We also notice that the ion part of the current, which should be very low, also shows an hysteresis. However, on this part of the signal, the measurement without plasma varies significantly: We assume that the amplitude of this response without plasma drifts over time, and that the subtraction of the signal with no plasma induces this asymmetry: We then correct the overall amplitude of the signal without plasma to limit this asymmetry. This factor correction is 20% maximum.

Delay and amplitude corrected signal

The current-voltage graph above shows the same signals corrected for delay ($0.47\,\mu s$) and amplitude (-4.5%).

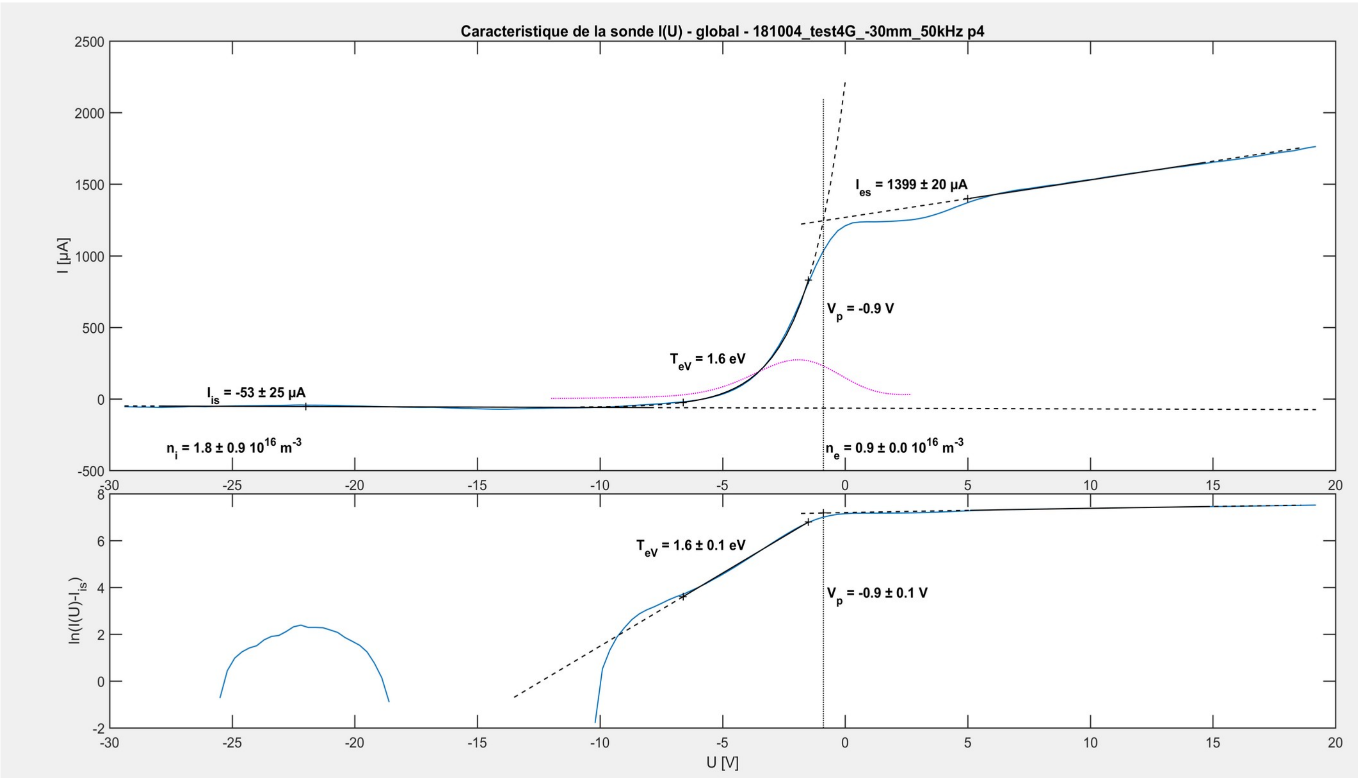
The mean hysteresis is much less pronounced than before the correction.



2.4 Mean characteristics

A current-voltage characteristic analysis can be carried out on the time mean curve. The figure below shows the analysis result.

The upper graph shows the data (in blue) in linear scale. The lower graph shows the electronic part of the electron current in logarithmic scale.



As expected, the (positive) electron current is stronger than the ion current.

In order to estimate the electron saturation current, we restrict the analysis over a few volts above the maximum expected plasma potential (in the present case $5\,V$). In order to estimate this current, the experimental data are adjusted by a affine function over a wide voltage range ($5\,V$ to $15\,V$, black plot on the graph). The saturation is not independent of the probe voltage: the greater the difference between the probe and plasma potentials are, the thicker the probe sheath is, and the greater the saturation current is. The saturation current is estimated for the lowest voltage of this range ($5\,V$ in this case).

In order to characterize the ion saturation current, we assume that below a certain voltage ($-22\,V$ in this case), all electrons are repelled and only the ion saturation current persists. The measurement data are adjusted over a certain voltage range (in this case $-28\,V$ to $-22\,V$) by a affine function, and the value of the ion saturation current is considered for a given voltage (in this case $-22\,V$).

In order to estimate the electron temperature, we first subtract the ion current from the total current using the adjustment obtained for the ion saturation current. The result is shown in logarithmic scale (the lower graph on the figure).

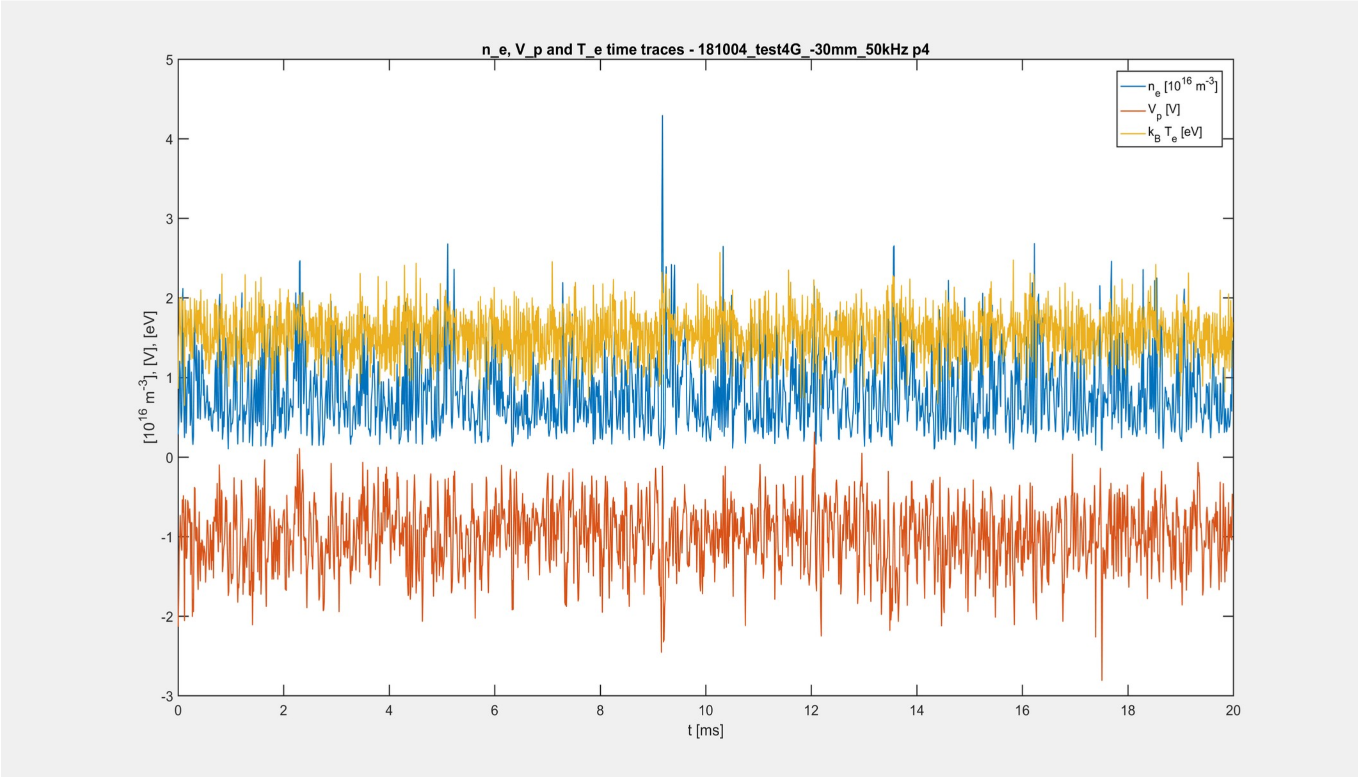
The electron current exponential growth appears as a a linear growth in logarithmic scale. The voltage range over which this growth takes place is unknown a priori. The upper limit of the exponential growth range is characterized by the fact the current derivative with respect to the voltage (in purple on the graph), increases in the exponential part and then decreases because of the saturation: the derivative maximum corresponds to the exponential growth range upper limit. A first affine adjustment is done around this derivative maximum over a $1.5\,V$ range. Since the affine trend of the characteristic logarithm appears to be over a larger voltage range, the voltage range is extended as long as the deviation between the data and the affine adjustment does not increase rapidly. Once this voltage range is optimized, the electron temperature is deduced from the inverse of the affine function slope. The plasma potential is estimated by looking for the intersection between the affine adjustment and the saturation affine adjustment.

Knowing the electron temperature and of the plasma potential, we can deduce the ion and electron densities from the ion and electron saturation currents.

2.5 Time dynamics of plasma parameters

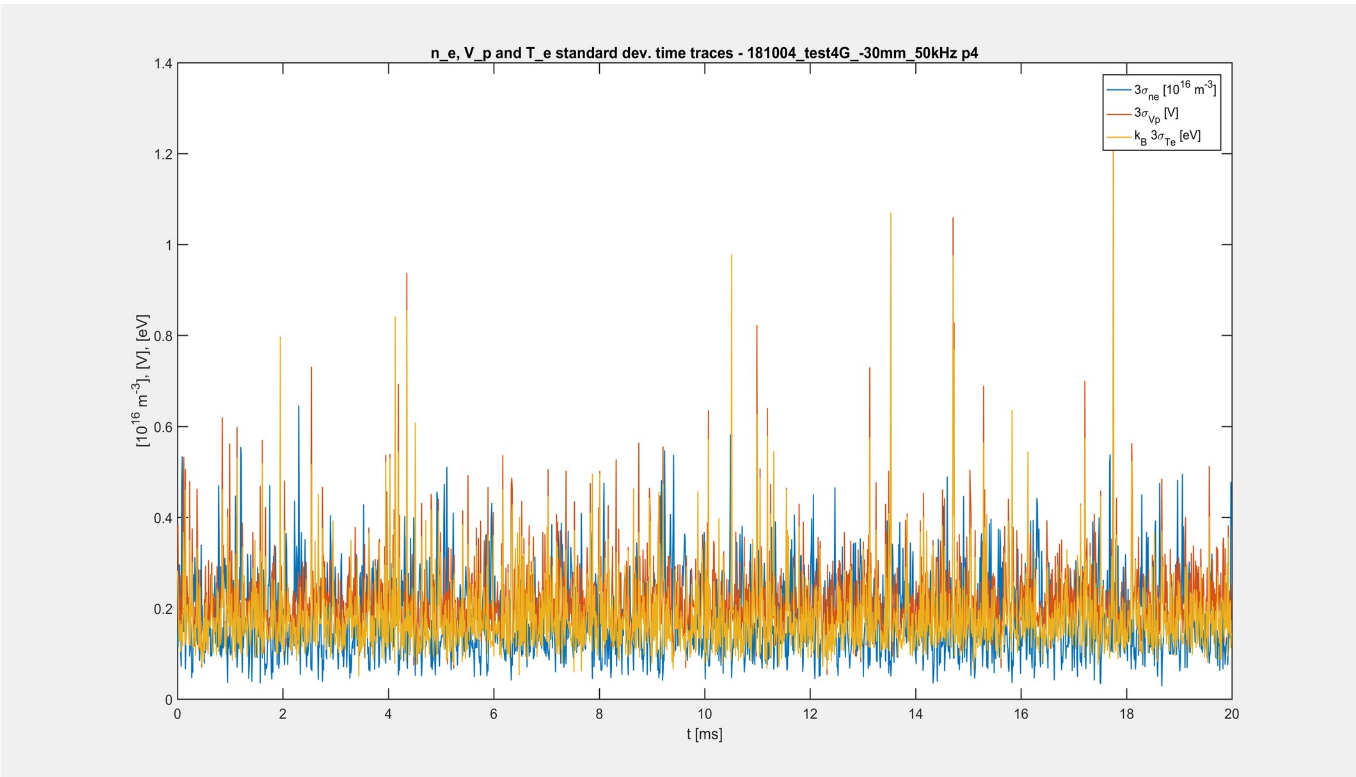
The polarization frequency ($f_g=50\,kHz$) is large enough here, to process the characteristic on each front (each blue part on the voltage-current graph above), with no time averaging: the plasma changes sufficiently slowly over the time of a generator front ($10\,\mu s$) to deform the curve. We then apply the processing of the current-voltage characteristic on each generator half-period. The time signals of the electron density, of the plasma potential and of the electron temperature can be extracted.

The graph below shows the discrete time data reconstructed by this processing for these 3 parameters.



We can extract these signals at the frequency of $f = 100\ kHz$ (2 values for each generator period).

In order to ensure the uncertainty on the main plasma parameter evaluation is satisfactory, the time variation of the uncertainty of each of these parameters over time is also estimated. These uncertainties are evaluated for each characteristic, from the differences between the measurement data and the affine adjustments made for the analysis. These uncertainties are estimated at three times the standard deviation.



3 Sheath model

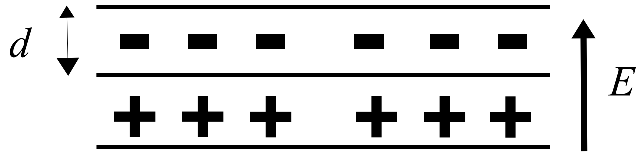
In order to determine the ion and electron flows at the surface of the sheath, it is necessary to determine the behavior of the ions and the electrons in the sheath.

3.1 Preliminary notions

Before developing the sheath model, we study some important notions in order to understand the behavior of the sheath.

1.a Debye length

The sheath around the probe is characterized by significant potential variations and charge separations. What is the typical charge separation distance?



We consider a plasma composed of positive (ions) and negative (electrons) opposite charges of the same density $n_i = n_e$. We assume a plane charge separation at a depth d between both species.

The charge per unit area is:

$$q = d\, n_e q_e \ .$$

This charge separation induces an electric field E :

$$E = \frac{d\, n_e q_e}{\varepsilon_0} \ .$$

The electrical force per unit area, between positive and negative charges, is:

$$F = -q\, E = \frac{-d^2 n_e^2 q_e^2}{\varepsilon_0} \ .$$

The electrical potential energy per unit area corresponding to this charge separation is :

$$E_q = d\, F = \frac{-d^3 n_e^2 q_e^2}{\varepsilon_0} \ .$$

The electron internal kinetic energy per unit area, when the electron temperature is T_e , is :

$$E_i = d\, n_e k_B T_e \ .$$

The maximum charge separation distance allowed by a plasma with this internal energy corresponds to the case:

$$|E_i| = |E_q| \ .$$

The resulting distance is the electron Debye length λ_{De} :

$$\lambda_{De} = d = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_e q_e^2}} \ .$$

Typically, for an electron density $n_e = 10^{17}\, m^{-3}$ and an electron temperature

$T_e = 2\,eV$, the electron Debye length is $\lambda_{De} = 33\,\mu m$.

The Debye length is the characteristic length scale for direct electric effects.

1.b Child–Langmuir law

The Child-Langmuir law shows there is a limit on the surface current of a charged particle beam propagating in vacuum between 2 surfaces at different potentials, with no initial kinetic energy. We study how this law has consequences on the sheath shape.

The problem is unidirectional. Only one particle species is present.

Particles have a negative charge $-q_s < 0$. On the emission surface, at $x = 0$, the particle velocity is zero $v_s(0) = 0$. This surface is the potential reference: $U(0) = 0$. We also assume the electric field is zero on this emission surface: $\frac{d}{dx} U(0) = 0$. On the second surface at the distance $x = d$, the potential is positive, in order to attract negative charges: $U(d) = U_d > 0$.

The Poisson equation is applied to the beam:

$$\frac{d^2}{dx^2} U = \frac{q_s n_s(x)}{\varepsilon_0} \ .$$

The particle velocity is determined by the particle energy conservation:

$$\frac{1}{2} m_s v_s(x)^2 = q_s U(x) \ .$$

The particle velocity depends on the potential:

$$v_s(x) = \left(\frac{2 q_s U(x)}{m_s} \right)^{\frac{1}{2}} \ .$$

Since the particle flux (flow per unit area) is conserved:

$$j_s = n_s(x) v_s(x) \ ,$$

the density depends on the potential:

$$n_s(x) = j_s \left(\frac{m_s}{2 q_s U(x)} \right)^{\frac{1}{2}} \ .$$

The Poisson equation becomes a quadratic equation on the potential U :

$$\frac{d^2}{dx^2} U = \frac{q_s j_s}{\varepsilon_0} \left(\frac{m_s}{2 q_s U} \right)^{\frac{1}{2}} \ .$$

In order to be able to integrate the members of the equation, we multiply the 2 members by $\frac{d}{dx} U$, and we integrate following x :

$$\int_0^x \frac{d}{dx} U \frac{d^2}{dx^2} U \, dx = \int_0^x \frac{q_s j_s}{\varepsilon_0} \left(\frac{m_s}{2 q_s} \right)^{\frac{1}{2}} U^{\frac{-1}{2}} \frac{d}{dx} U \ .$$

At the initial surface, the conditions are $U(0) = 0$ and $\frac{d}{dx} U(0) = 0$:

$$\left(\frac{d}{dx} U \right)^2 = 2 \frac{j_s}{\varepsilon_0} (2 q_s m_s)^{\frac{1}{2}} U^{\frac{1}{2}}$$

the square root of the equation is:

$$U^{\frac{-1}{4}} \frac{d}{dx} U = \left(2 \frac{j_s}{\varepsilon_0}\right)^{\frac{1}{2}} (2 q_s m_s)^{\frac{1}{4}} \quad .$$

This equation can be integrated (with the limit condition $U(0)=0$) :

$$U^{\frac{3}{4}} = \frac{3}{2} \left(\frac{j_s}{\varepsilon_0}\right)^{\frac{1}{2}} \left(\frac{q_s m_s}{2}\right)^{\frac{1}{4}} x \quad .$$

This relation is applied to the second surface, at $x=d$:

$$U_d^{3/2} = \frac{9}{4} \frac{j_s}{\varepsilon_0} \left(\frac{q_s m_s}{2}\right)^{\frac{1}{2}} d^2 \quad .$$

This condition determines the particle flux :

$$j_s = \frac{4}{9} \varepsilon_0 \left(\frac{2}{q_s m_s}\right)^{\frac{1}{2}} \frac{U_d^{3/2}}{d^2} \quad .$$

The particle flux in a particle beam between 2 plates at 2 different potentials is limited by the potential difference and the distance between the plates.

As the conditions in the probe sheath are close to those of the Child-Langmuir law, comparable relation determines the sheath depth the potential difference between the plasma and the probe.

1.c Collision mean free paths

In order to assess the effect of collisions inside the sheath, we compare the mean free paths for different particles at the characteristic scale of the sheath, the Debye length.

Collisions of charged particle on neutrals

The mean free path l_{ns} particle species s , ion or electron, on the neutrals n (density n_n), is given by :

$$l_{ns} = \frac{1}{\sigma_{ns} n_n} \quad .$$

σ_{ns} is the interaction cross section between the species s and the neutrals.

For argon, the cross section of the ions Ar^+ on the neutrals is $\sigma_{\mathrm{ni}} = 10^{-18} m^2$, and that on electrons is $\sigma_{\mathrm{ne}} = 10^{-19} m^2$. For typical neutral densities of the order of $n_n = 10^{19} m^{-3}$, these mean free paths are $l_{ni} = 10\, cm$ for ions, and $l_{ne} = 1\, m$ for electrons. These distances are large compared to the probe sheath thickness, which is of the order of the Debye length λ_{De} , much smaller.

Collisions between charged particles

The mean free path between charged particles, here electrons, is estimated from Coulomb interactions, taking into account the screening effect of charged particles by their neighbors.

The collision cross section for electron Coulomb interaction is given by (b is the impact distance) :

$$\sigma_{ee} = \left(\frac{q_e^2}{4 \pi \varepsilon_0}\right)^2 \frac{4 \pi}{m_e^2 v_e^4} \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

With no impact distance limit, the Coulomb cross section diverges. Direct Coulomb interaction are limited by the Debye length $b_{max}=\lambda_{De}$. The usual lower limit is given by the impact distance for which the scattering angle is 1 : $b_{min}=\frac{q_e^2}{4\pi\varepsilon_0}\frac{1}{m_e v_e^2}$.

This integral known as the Coulomb logarithm is usually of the order of 10 :

$$\ln\Lambda=\int_{b_{min}}^{b_{max}}\frac{db}{b}\sim 10 \ .$$

We the electron velocity with their thermal value $k_B T_e=m_e v_e^2$

$$\sigma_{ee}=\left(\frac{q_e^2}{4\pi\varepsilon_0}\right)^2\frac{4\pi}{k_B^2 T_e^2}\ln\Lambda$$

Using the expression of the Debye length :

$$\sigma_{ee}=\frac{1}{4\pi n_e^2\lambda_{De}^4}\ln\Lambda$$

This mean free path can be approximated by:

$$l_{ee}=\frac{4\pi}{\ln\Lambda}n_e\lambda_{De}^4 \ .$$

l_{ee} is large compared to the typical probe sheath depth λ_{De} , if the particle number in the Debye sphere ($n_e\lambda_{De}^3$) is large.

For the typical electron density $n_e=10^{17}m^{-3}$, and the Debye length $\lambda_{De}=33\,\mu m$, this number is several thousand : $n_e\lambda_{De}^3\sim 3\ 10^3$.

For the typical conditions, the collisions between charged particles are negligible at the scale of the probe sheath thickness: $l_{ee}=10\,cm$.

3.2 Ion sheath around the probe

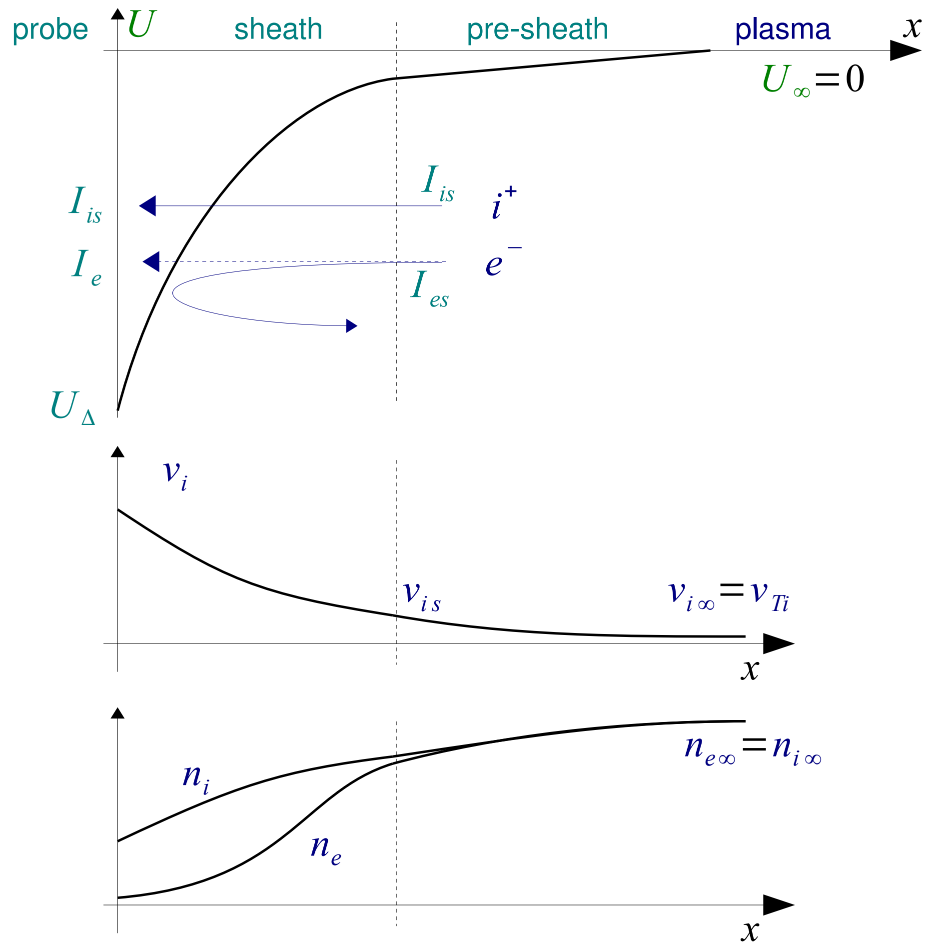
We consider the Langmuir probe is a completely absorbent conductive surface immersed in the plasma: electrons are absorbed by the probe and ions touching the probe are recombined.

Since the Debye length is small compared to the size of the probe, we consider that the problem can be approximated in a Cartesian geometry. The surface of the probe is flat. The problem is unidirectional: the parameters are considered uniform in the planes parallel to the surface of the probe. Velocities components parallel to the probe do not play any role. The parameters vary only in the direction perpendicular to the surface of the probe, $\vec{e}_x$.

The electric field due to the potential difference between the probe and the plasma has different effects on the ions and electrons in the sheath. In the sheath, quasi-neutrality is not respected. This zone is a few Debye lengths thick.

For the ion sheath, the probe potential is lower than the surrounding plasma potential. The probe attracts ions and repels electrons.

2.a Sheath and pre-sheath



Sheath boundary conditions with the probe and on the plasma side

Far from the sheath, in the plasma, the plasma potential is zero: $U_\infty = 0$. In order to simplify the expressions, the plasma potential is the reference.

The probe potential is lower than the plasma potential. The sheath potential is negative everywhere: $U \leq 0$.

The probe potential is U_Δ ($U_\Delta = U_L - U_p < 0$).

Far from the probe, the plasma is almost neutral, the electron density $n_{e\infty}$ and the ion density $n_{i\infty}$ are almost equal :

$$n_{e\infty} = n_{i\infty} \quad .$$

The ion temperature T_i and electron temperature T_e are uniform in plasma around the probe. The ion temperature is very low compared to the electron temperature: $T_i \ll T_e$.

Sheath and pre-sheath

The main force between the probe and the plasma is the electric force: the Poisson equation characterizes the particle behavior in the sheath. We show there is no solution for the Poisson equation with the plasma conditions on the plasma side: it is necessary to introduce a second zone beyond the sheath, in order to be able to connect the boundary conditions on the plasma side. This is the pre-sheath. This zone is characterized by the fact that this zone remains sensitive to the potential imposed by the probe, but the electrical forces are not dominant there (i.e. the Poisson equation is no longer sufficient to describe the processes).

2.b Particle behavior in the ion sheath

We first study the sheath.

In the sheath, quasi-neutrality is no longer respected: the dynamics between ions and electrons follows Poisson's law :

$$\frac{d^2}{dx^2}U=\frac{q_e}{\varepsilon_0}(n_e-n_i)$$

We neglect the other effects (collisions, ionization...) in the sheath.

Behavior of electrons

The electron energy distribution function in the sheath f_e depends on the distribution they had in the plasma away from the probe $f_{e\infty}$. At position x where the potential is U , the electrons whose initial energy away in the plasma $E_{ce\infty}$ is smaller than $q_e U$ are repelled before reaching this position.

The relation between the energy inside and outside the sheath for the non repelled electrons is :

$$E_{ce}=E_{ce\infty}+q_e U \quad .$$

The particle conservation (for the non repelled electrons) between the outside of the sheath and the sheath is given by the relation :

$$f_e(E_{ce})\,dE_{ce}=f_{e\infty}(E_{ce\infty})\,dE_{ce\infty} \quad .$$

Because the energy relation is only a uniform subtraction :

$$dE_{ce}=dE_{ce\infty} \quad .$$

As a result, the shape of the energy distribution is preserved :

$$f_e(E_{ce})=f_{e\infty}(E_{ce\infty}) \quad .$$

The electron kinetic energy distribution far from the probe is a Maxwellian, at the density $n_{e\infty}$ and the temperature T_e :

$$f_{e\infty}(E_\infty)=\frac{n_{e\infty}}{k_B Te}e^{\frac{-E_\infty}{k_B Te}} \quad .$$

The electron energy distribution in the sheath is deduced:

$$f_e(E)=\frac{n_{e\infty}}{k_B Te}e^{\frac{-(E-q_e U)}{k_B Te}} \quad .$$

The shape is also a Maxwellian with the same temperature T_e :

$$f_e(E)=\frac{n_{e\infty}}{k_B Te}e^{\frac{q_e U}{k_B Te}}e^{\frac{-E}{k_B Te}} \quad .$$

The electron density in the sheath varies with the potential U :

$$n_e=\int_0^\infty f_e(E)\,dE=n_{e\infty}e^{\frac{q_e U}{k_B Te}} \quad .$$

The electron density is reduced by loss of repelled electrons.

Ion behavior

We will verify that the Poisson equation applied in the sheath does not find an adequate solution with an ion thermal velocity at the sheath boundary as low as the thermal ionic speed. At the boundary of the sheath, the ions have a velocity v_{is} larger than the ion thermal velocity. The plasma potential is U_s , non zero, because of the pre-sheath. The parameter values at the sheath boundary are established below.

Inside the sheath, the ions are accelerated due to the electric field:

$$\frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i v_{is}^2 - q_e(U - U_s) \ ,$$

or :

$$v_i = \sqrt{\frac{\frac{1}{2}m_i v_{is}^2 - q_e(U - U_s)}{\frac{1}{2}m_i}} \ .$$

Ion flux per unit area, j_i is conserved through the sheath :

$$j_i = n_i v_i = j_{is} = n_{is} v_{is} \ .$$

These conditions determine the ion density as a function of the sheath potential :

$$n_i = n_{is}\frac{v_{is}}{v_i} = n_{is}\sqrt{\frac{\frac{1}{2}m_i v_{is}^2}{\frac{1}{2}m_i v_{is}^2 - q_e(U - U_s)}} \ .$$

We assume the plasma is quasi-neutral in the pre-sheath :

$$n_{is} \sim n_{es}$$

with :

$$n_{es} = n_{e\infty} e^{\frac{q_e U_s}{k_B Te}} \ .$$

Poisson equation

The above expressions for the electron and ion densities n_e and n_i are used:

$$\frac{d^2}{dx^2}U = \frac{q_e}{\varepsilon_0} n_{es}\left(e^{\frac{q_e(U - U_s)}{k_B Te}} - \sqrt{\frac{\frac{1}{2}m_i v_{is}^2}{\frac{1}{2}m_i v_{is}^2 - q_e(U - U_s)}} \right) \ .$$

This equation on the potential, associated with the boundary conditions on the probe and on the pre-sheath side, is sufficient to determine the sheath potential profile. But this equation has no analytical solutions.

2.c Connection between sheath and pre-sheath

Bohm criterion

What is the simplified form of this equation near the plasma sheath boundary (where the potential is close to U_s) ?

We assume the potential energy is much smaller than the electron and ion kinetic energies:

$$-q_e(U - U_s) \ll k_B Te \ ,$$

$$-q_e(U-U_s) \ll \frac{1}{2} m_i v_{is}^2 \quad .$$

The equation is simplified :

$$\frac{d^2}{dx^2} U = \left(1 - \frac{k_B Te}{m_i} \frac{1}{v_{is}^2}\right) \frac{U-U_s}{\lambda_{Des}^2} \quad ,$$

where λ_{Des} is the electron Debye length at the boundary :

$$\lambda_{Des} = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_{es} q_e^2}} \quad .$$

If the ion velocity at the sheath boundary verifies :

$$v_{is} < \sqrt{\frac{k_B Te}{m_i}} \quad .$$

The solution of the simplified equation at low potential oscillates spatially: this solution can not be connected with the sheath inner zone where the potential varies in a monotonous way, with a weak gradient in the direction of the plasma.

In order to get a realistic solution for the whole sheath, the ion velocity at the sheath boundary should be larger than a specific velocity:

$$v_{is} \geq \sqrt{\frac{k_B Te}{m_i}} \quad .$$

This is the Bohm criterion.

The sheath potential no longer oscillates. The potential profile is close to a monotonic, quasi-exponential shape over the sheath.

The Bohm criterion expresses the fact that the ion density close to the sheath boundary:

$$n_i \sim n_{is} \left(1 - \frac{q_e(U-U_s)}{m_i v_{is}^2}\right)$$

should always be larger than the electron density :

$$n_e \sim n_{es} \left(1 - \frac{q_e(U-U_s)}{k_B Te}\right) \quad .$$

in order to get a monotonous potential profile.

The ion density decreases less rapidly when approaching the probe than the electron density, the ion velocity at the sheath boundary must not be too low.

The sheath boundary corresponds to the condition fixed by the Bohm criterion. The ion sheath boundary velocity v_{is} is :

$$v_{is} = \sqrt{\frac{k_B Te}{m_i}} \quad .$$

This ion velocity is the Bohm velocity (it is also the plasma ion acoustic velocity).

The potential at the sheath boundary, U_s is deduced from the ion velocity by the law of ion energy transfer:

$$\frac{1}{2} m_i v_{is}^2 = \frac{1}{2} m_i v_{i\infty}^2 - q_e U_s \quad .$$

The ion velocity far in the plasma is of the order of the ion thermal velocity:

$$v_{i\infty}=\sqrt{\frac{\gamma_i k_B T_i}{m_i}} \quad .$$

The relation for U_s is :

$$\frac{1}{2}k_B T_e=\frac{1}{2}\gamma_i k_B T_i-q_e U_s \quad .$$

Since the ion temperature is much smaller than the electron one, $T_i \ll T_e$, the sheath boundary potential U_s mostly depends only on electron temperature :

$$U_s=-\frac{k_B Te}{2 q_e} \quad .$$

Ion flux at the sheath surface

Beyond the sheath boundary, the Poisson equation no longer has a stable solution with the sheath because of the Bohm criterion. The electric force no longer dominates the ion and electron dynamics. Beyond the sheath boundary, quasi-neutrality applies. The ion density is equal to the electron density. The electron density depends on the plasma potential:

$$n_{is}=n_{es}=n_{e\infty} e^{\frac{q_e U_s}{k_B Te}}=n_{e\infty} e^{-1/2} \quad .$$

The ion flux across the sheath boundary is:

$$j_{is}=n_{is} v_{is}=n_{e\infty} e^{-1/2}\sqrt{\frac{k_B Te}{m_i}} \quad .$$

The ion current on the probe is determined by this ion flux across the sheath surface and the sheath surface itself.

Pre-sheath

The sheath boundary conditions on v_{is} and U_s are different from those corresponding to plasma in the absence of a probe : $v_{i\infty}\sim 0$ and $U_\infty=0$. The pre-sheath is characterized by the slow acceleration of the ions by the slow variation of the potential. In the pre-sheath, the Poisson equation alone does not have a solution compatible with the sheath: the acceleration does not occur directly through the electric field due to the potential difference between the sheath boundary and the plasma. In this zone, other effects (ionization, collisions, etc.) must intervene to control this acceleration.

Since the plasma is quasi-neutral in this zone, the pre-sheath thickness is not of the order of magnitude of the Debye length. The spatial extension of the pre-sheath might be much larger than the sheath. When collisions prevail, the pre-sheath size of the order of the ion collision mean free paths.

If the size of the pre-sheath is larger than the probe size, the pre-sheath rather has a spherical shape: The particle conservation no longer induce a conservation of the ion surface fluxes.

2.d Ion sheath thickness

In order to determine the probe collection surface, the probe sheath surface must be determined.

We consider the case where the probe potential is in absolute value much larger than the electron thermal energy divided by the elementary charge q_e :

$$|U_\Delta| \gg \frac{k_B Te}{q_e} \quad .$$

In this case, the sheath electron density is neglected with respect to the ion density :

$$n_e \ll n_i \quad .$$

We apply this condition to the whole sheath. At the sheath boundary, the electron density reaches that of the ions: the approximation is not valid. But this problematic area is small compared to the rest of the sheath: it is neglected.

The ion velocity in the sheath is:

$$v_i = \sqrt{\frac{-2 q_e U}{m_i}} \quad .$$

The Poisson equation in the sheath is then :

$$\frac{d^2}{dx^2} U = \frac{q_e}{\varepsilon_0} j_i \sqrt{\frac{m_i}{-2 q_e U}} \quad .$$

The equation is the same as the Child-Langmuir law situation (only one species), with different boundary conditions between the sheath and the pre-sheath.

To solve the Poisson equation, we multiply the 2 members by $\frac{d}{dx} U$, and we integrate along x :

$$\int_{x_s}^{x} \frac{d}{dx} U \frac{d^2}{dx^2} U \, dx' = \int_{x_s}^{x} \frac{q_e}{\varepsilon_0} j_i \sqrt{\frac{m_i}{-2 q_e U}} \frac{d}{dx} U \, dx' \quad ,$$

where x_s is the sheath boundary position.

At this surface, the potential is $U(x_s) = U_s$ and the electric field is almost zero, $\frac{d}{dx} U(x_s) = 0$:

$$\left(\frac{d}{dx} U\right)^2 = \frac{2 j_i}{\varepsilon_0} \sqrt{2 q_e m_i} \left(\sqrt{-U} - \sqrt{-U_s}\right) \quad .$$

This equation is integrated a second time to obtain a relation between the potential U and position x :

$$\left(\sqrt{-U} - \sqrt{-U_s}\right)^{1/2} \left(\sqrt{-U} + 2\sqrt{-U_s}\right) = \frac{3}{4} \left(\frac{8 j_i^2 m_i q_e}{\varepsilon_0^2}\right)^{1/4} (x_s - x) \quad .$$

We apply this result to the probe, $x = 0$, with the probe potential, U_Δ . The sheath boundary potential is :

$$U_s = -\frac{k_B Te}{2 q_e} \quad ,$$

and the ion flux at the sheath boundary is :

$$j_{is} = n_{is} v_{is} = n_{e\infty} e^{-1/2} \sqrt{\frac{k_B Te}{m_i}} \quad .$$

The equation introduces the Debye length :

$$\lambda_{De} = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_e q_e^2}} \quad .$$

The relation links the sheath boundary position x_s to the probe potential U_Δ . The sheath boundary position corresponds to the sheath thickness:

$$e_{is}(U_\Delta)=x_s=\frac{2}{3}(2e)^{1/4}\left(\sqrt{\frac{-q_e U_\Delta}{k_B T_e}}-\frac{1}{\sqrt{2}}\right)^{1/2}\left(\sqrt{\frac{-q_e U_\Delta}{k_B T_e}}+\sqrt{2}\right)\lambda_{De} \quad .$$

Since the probe potential is much larger than the electron thermal energy divided by q_e :

$$|U_\Delta|\gg\frac{k_B Te}{q_e} \quad ,$$

the expression is simplified :

$$e_{is}(U_\Delta)=\frac{2}{3}(2e)^{1/4}\left(\frac{-q_e U_\Delta}{k_B T_e}\right)^{3/4}\lambda_{De}$$

The sheath thickness is of the order of a few times the Debye length.

3.3 Electron sheath

In the case where the probe potential U_Δ is higher than the plasma potential, a sheath also forms of positive potential: the electrons are more numerous than the ions. For this sheath, part of the ions is repelled:

$$U_\Delta\geq 0 \quad .$$

3.a Electron flux on the probe

Electron flux

Since the electron thermal velocity is large in the plasma, the Bohm criterion at sheath boundary is always satisfied: the sheath boundary parameters have the same values as further in the plasma : there is no pre-sheath.

The electrons flux reaching the probe is the product of the electron density by the mean electron velocity in the plasma, at the sheath boundary. But only half of the electrons should be taken into account, because the other half has a velocity oriented in the opposite direction to the surface of the sheath.

For a Maxwellian velocity distribution (in modulus):

$$f_e(v)=\left(\frac{m_e}{2\pi k_B T_e}\right)^{3/2} 4\pi v^2 e^{\frac{-m_e v^2}{2k_B T_e}} \quad .$$

The mean velocity modulus is :

$$v_{e\infty}=\int_0^\infty f_e(v)v\,dv \quad .$$

The velocity integrated by substitution, $u=v^2$:

$$\int_0^\infty v^3 e^{-bv^2}dv=\int_0^\infty \frac{u}{2}e^{-bu}du \quad ,$$

and integration by parts :

$$\int_0^\infty v^3 e^{-bv^2}dv=\left[\frac{-u}{2b}e^{-bu}\right]_0^\infty-\int_0^\infty\frac{-1}{2b}e^{-bu}du=0-\left[\frac{1}{2b^2}e^{-bu}\right]_0^\infty=\frac{1}{2b^2} \quad .$$

The mean velocity modulus :

$$v_{e\infty}=\int_0^\infty f_e(v)v\,dv=\sqrt{\frac{8k_B T_e}{\pi m_e}} \quad .$$

This mean velocity modulus is different from the mean velocity component along the direction normal to the probe. The Maxwellian distribution in a Cartesian coordinate system is factorized along the 3 directions :

$$f_e(v_x, v_y, v_z) = f_{ex}(v_x) f_{ex}(v_y) f_{ex}(v_z) \ .$$

with a common unidirectional distribution :

$$f_{ex}(v_x) = \left(\frac{m_e}{2\pi k_B T_e}\right)^{1/2} e^{\frac{-m_e v_x^2}{2 k_B T_e}} \ ,$$

with $\int_{-\infty}^{\infty} f_{ex}(v_x)\, dv_x = 1$.

Due to the distribution factorization, the mean velocity along one axis is independent of the other axes :

$$v_{ex\infty} = \int_0^\infty dv_x f_{ex}(v_x) v_x = \left(\frac{m_e}{2\pi k_B T_e}\right)^{1/2} \left[\frac{-k_B T_e}{m_e} e^{\frac{-m_e v_x^2}{2 k_B T_e}}\right]_0^\infty = \sqrt{\frac{k_B T_e}{2\pi m_e}} = \frac{1}{4} v_{e\infty} \ .$$

The mean axial velocity is a quarter of the mean velocity modulus.

The electron flux crossing the sheath boundary is then :

$$j_{es} = \frac{1}{4} n_{e\infty} \sqrt{\frac{8 k_B T_e}{\pi m_e}} \ .$$

Ion flux

The ions present in the electron sheath have a behavior comparable to the electrons in the ion sheath. Their density evolves in a comparable way due to the energy loss.

$$n_i = n_{e\infty} e^{\frac{-q_e U}{k_B Ti}} \ .$$

Since the ion temperature is very low, as soon as the probe potential is slightly higher than the plasma potential, the ion flux is negligible:

$$j_{is} = 0 \ .$$

3.b Electron Sheath Thickness

In order to evaluate the electron sheath thickness, we use hypotheses analogous to those used for the ion sheath. The Child-Langmuir law conditions are satisfied. The equation governing the potential is the same, but the boundary conditions at the sheath boundary differ.

We suppose the probe potential is sufficiently negative (the electric potential energy is much larger than the kinetic energy at the sheath boundary:

$$U_\Delta \gg \frac{k_B Te}{q_e}$$

The electron density can then be neglected compared to the ion density :

$$n_i \ll n_e \ .$$

The electron thermal velocity at the sheath boundary is neglected in comparison with their velocity inside the sheath. The electron velocity inside the sheath is :

$$v_e = \sqrt{\frac{2 q_e U}{m_e}} \ .$$

The Poisson equation in the sheath is then written :

$$\frac{d^2}{dx^2} U = \frac{q_e}{\varepsilon_0} j_e \sqrt{\frac{m_e}{2 q_e U}} \ .$$

The equation is solved in the same way. The boundary condition is $U = 0$ (and not

$U = U_s$ as for ion sheath).

$$U^{3/4} = \frac{3}{4}\left(\frac{8 j_e^2 m_e q_e}{\varepsilon_0^2}\right)^{1/4}(x_s - x) \quad .$$

We apply this result to the probe position, $x = 0$, with the probe potential, U_Δ .

$$U_\Delta^{3/4} = \frac{3}{4}\left(\frac{8 j_e^2 m_e q_e}{\varepsilon_0^2}\right)^{1/4} x_s \quad .$$

We use the expression of the electron flux at the surface of the sheath:

$$j_e = \frac{1}{4} n_{e\infty} \sqrt{\frac{8 k_B Te}{\pi m_e}} \quad ,$$

and the electron Debye length :

$$\lambda_{De} = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_e q_e^2}} \quad .$$

The sheath surface positions x_s corresponds to the sheath thickness e_{es} :

$$e_{es}(U_\Delta) = \frac{2}{3}(4\pi)^{1/4}\left(\frac{-q_e U_\Delta}{k_B T_e}\right)^{3/4} \lambda_{De} \quad .$$

For the same potential absolute value, the electron sheath is slightly larger than the ion sheath.

Density estimation from the saturation current

The probe collection area depends on the probe sheath thickness, hence on the Debye length, hence on the electron density. But the collection area is necessary to evaluate the plasma density.

$$I_{eS} = q_e n_{es} v_{es} A_e(n_{es})$$

The equation to extract the electron density from the electron saturation current is difficult to resolve. Numerically, it is possible to do a recurrent evaluation of the electron density and the collection area, starting with no sheath. Since the collection area is relatively not very sensitive to the electron density, the recurrent evaluation converges rapidly.

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