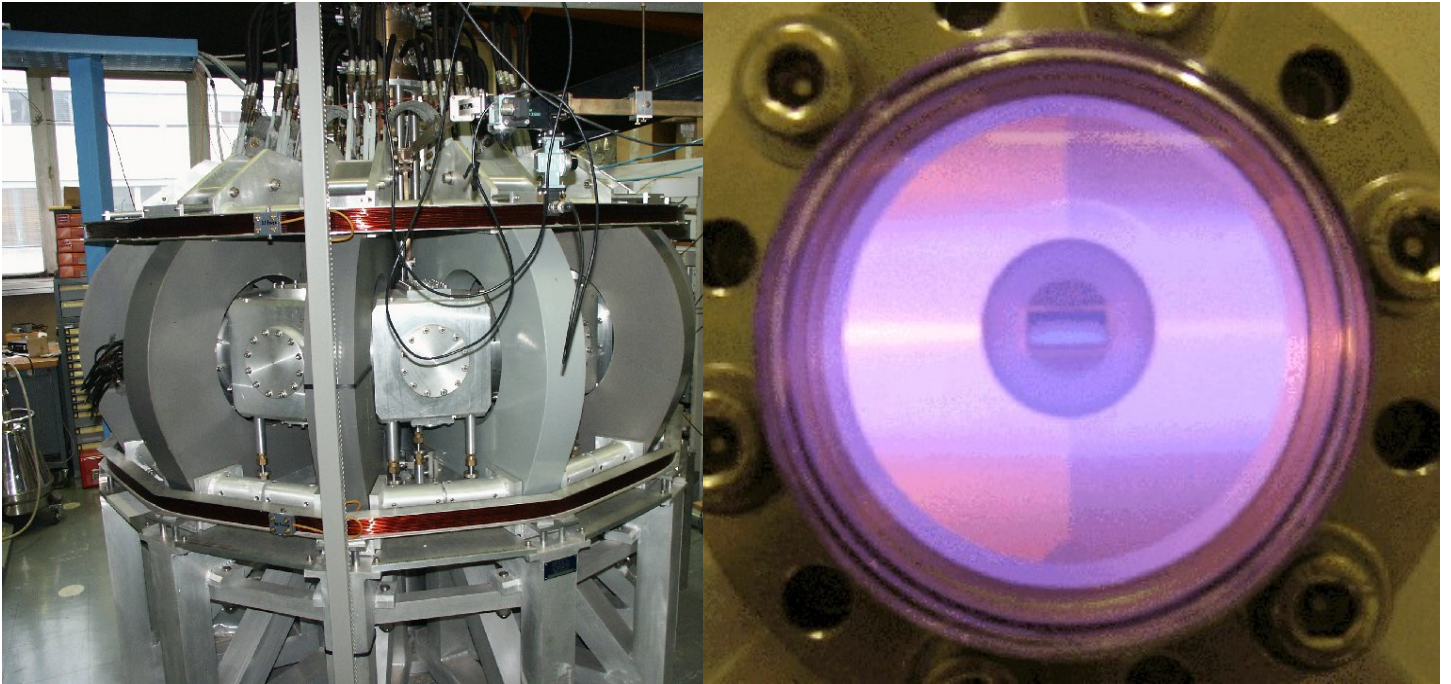


INTRODUCTION TO TORIC MAGNETIZED PLASMAS

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Plasma de Torix

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Introduction

Few properties of Torix magnetized plasmas are presented here.

- The cyclotron motion is perturbed by the particle drifts because the toric magnetic field is not uniform, and because of the presence of some induced electric field.
- Simple configuration toric magnetized plasmas are unstable because of these particle drifts.
- The addition of a small vertical magnetic field component reduces the effects of the particle drift due to the magnetic field toric geometry.
- The presence of a polarized filament induces a plasma poloidal rotation (relative to the torus small radius).

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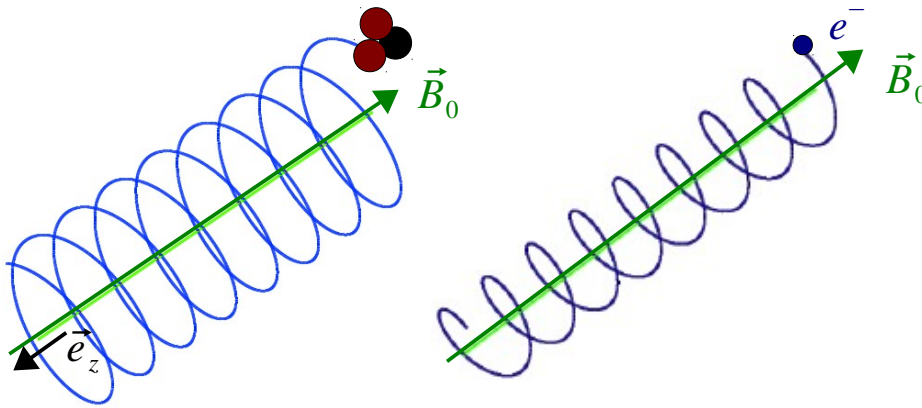
1 Toric Magnetized Plasma

1.1 Toric Configuration

We introduce the overall behavior of simple toric magnetized plasma. The plasma is created from Argon neutral gas. The gas is very partially ionized: the only charged species in the plasma are the ions, Ar^+ , and the electrons, e^- .

1.a Cyclotron Motion and Confinement

The magnetic confinement is based on the cyclotron motion of charged particles in a toric magnetic field created by currents in external coils. The magnetic field is intense enough so that the electron and ion Larmor radii are much smaller than the plasma size.



For an uniform magnetic field $\vec{B}_0 = B_0 \vec{e}_z$, and for the specie α (ion Ar^+ or electron e^-), with the charge q_α (the elementary charge q_e for Ar^+ , $-q_e$ for e^-) and the mass m_α , the charged particle motion equation is :

$$m_\alpha \vec{a}_\alpha = q_\alpha \vec{u}_\alpha \wedge \vec{B}_0 .$$

In the Cartesian coordinate system, the equations are :

$$a_{\alpha x} = \omega_{c\alpha} u_{\alpha y} ,$$

$$a_{\alpha y} = -\omega_{c\alpha} u_{\alpha x} ,$$

$$a_{\alpha z} = 0 ,$$

where $\omega_{c\alpha}$ is the cyclotron angular frequency:

$$\omega_{c\alpha} = \frac{q_\alpha B_0}{m_\alpha} .$$

The particle velocity solution is a rotation at constant frequency and modulus:

$$u_{\alpha x}(t) = u_{\perp\alpha} \cos[\omega_{c\alpha} t + \psi]$$

$$u_{\alpha y}(t) = -u_{\perp\alpha} \sin[\omega_{c\alpha} t + \psi]$$

$$u_{\alpha z}(t) = u_{\parallel\alpha} ,$$

where $u_{\perp\alpha}$ is the particle velocity perpendicular to the magnetic field, given by the initial conditions :

$$u_{\perp\alpha} = \sqrt{u_{\alpha x0}^2 + u_{\alpha y0}^2} ,$$

and $\vec{u}_{\parallel\alpha}$, the particle parallel velocity:

$$\vec{u}_{\parallel\alpha} = u_{\alpha z0} .$$

The solution for the particle position is:

$$x_{\alpha}(t) = x_{g\alpha} + \rho_{c\alpha} \sin[\omega_{c\alpha} t + \psi]$$

$$y_{\alpha}(t) = y_{g\alpha} + \rho_{c\alpha} \cos[\omega_{c\alpha} t + \psi]$$

$$z_{\alpha}(t) = z_{g\alpha}(t) = z_{g\alpha0} + u_{\parallel\alpha} t ,$$

where $\rho_{c\alpha}$ is the particle Larmor radius:

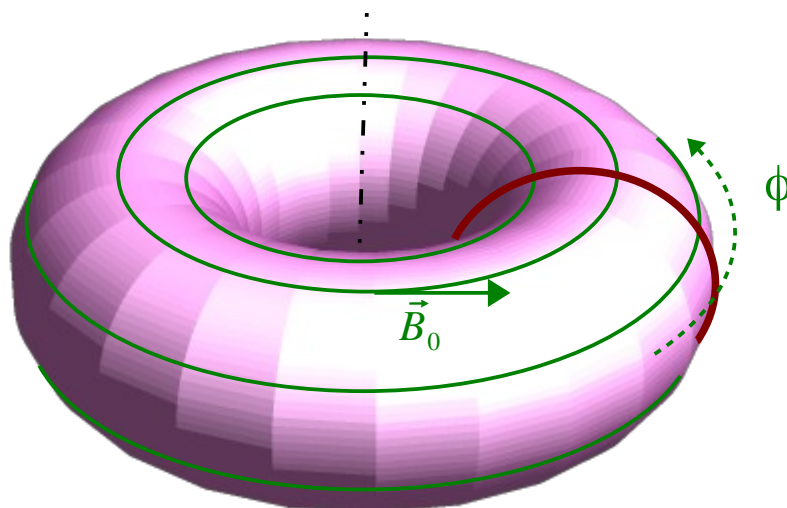
$$\rho_{c\alpha} = \frac{u_{\perp\alpha}}{\omega_{c\alpha}} .$$

$(x_{g\alpha}, y_{g\alpha}, z_{g\alpha}(t))$ is the guiding center trajectory. This is the particle position projection on the magnetic field line around which the particle is rotating. ψ is the initial cyclotron perpendicular velocity phase.

The cyclotron motion in an uniform magnetic field confine the particle motion only along 2 directions. The motion along the magnetic field lines is free : it is not modified by the magnetic field.

1.b Toric geometry

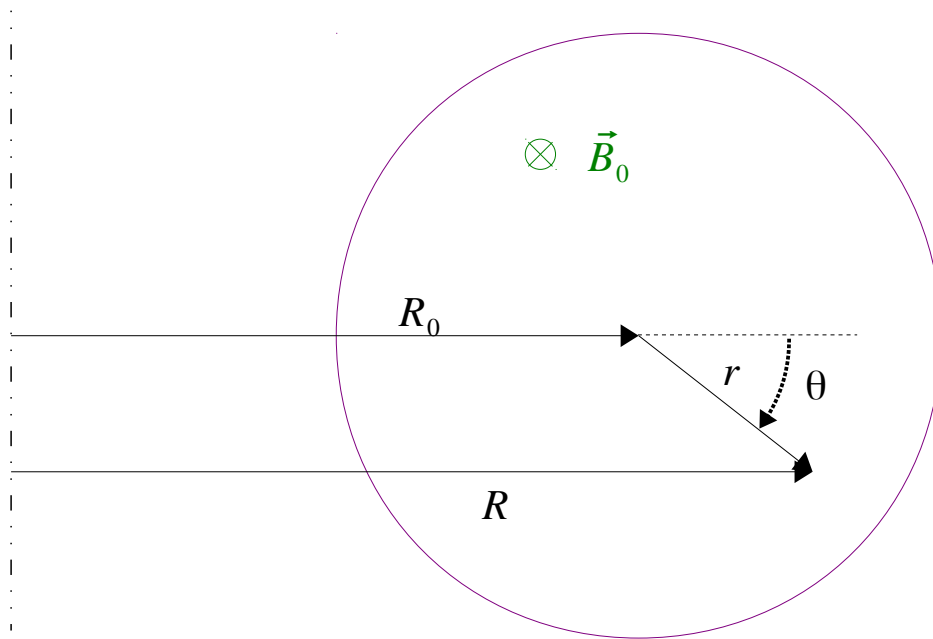
In order to limit particle losses along the magnetic field lines direction, the magnetic field lines are closed circles inside a tore. The magnetic field lines follow the main direction of the tore, known as the toroidal direction (green circle in the following figure). ϕ is the toroidal direction angle.



In first approximation, the plasma behavior is symmetrical along the toroidal direction.

Because of this symmetry, we restrict the plasma description to a poloidal cross section, perpendicular to the toroidal direction (corresponding to the red circle on the above figure).

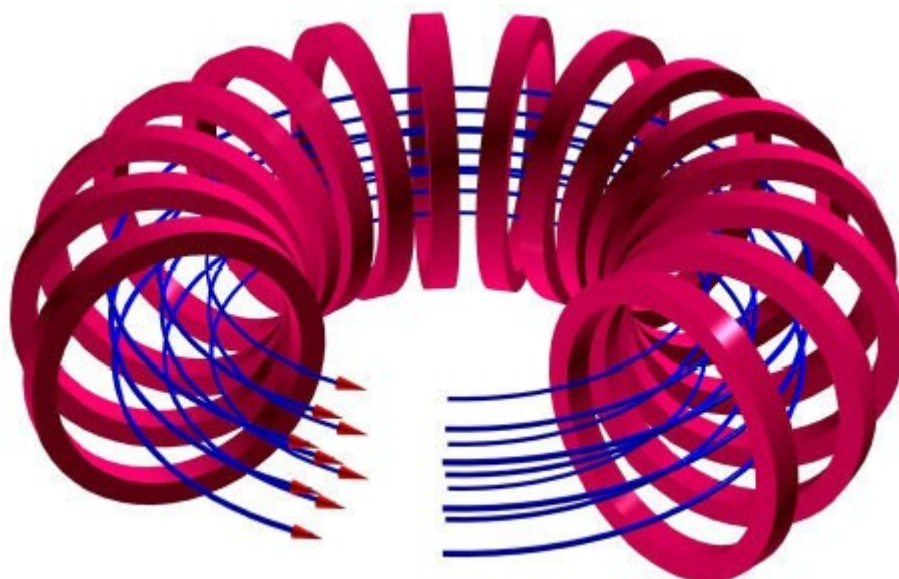
The following figure shows this poloidal section. The plasma torus is projected to a disc. The black dashed line on the left correspond to the position of the torus symmetry (central) axis :



For any position inside the torus, the large radius R is the distance between the position and the torus axis. In order to locate this position in the poloidal section, a polar reference frame is used. Its origin is the center of the torus section. The coordinates are the small radius r and the poloidal angle θ . R_0 is the torus section center large radius.

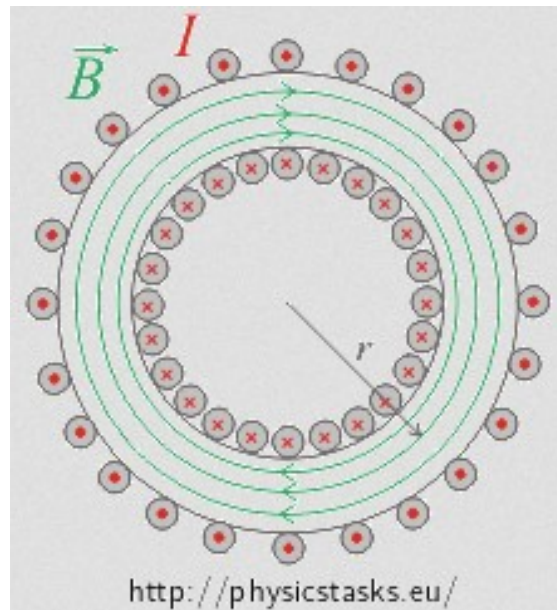
1.c Magnetic field configuration

The simpler magnetic field toric configuration is obtained using identical coils regularly placed around the torus. The currents through each coil are the same. The magnetic field toroidal symmetry depends on the number of coils around the torus.



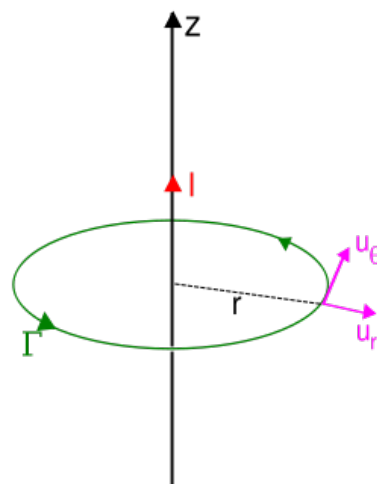
The magnetic field magnitude is not uniform along the large radius direction.

We consider the horizontal torus median section. The coil currents are opposite between the inner side and the outer side of the torus.



In order to evaluate the magnetic field magnitude inside the torus, the Ampere theorem is applied for each large radius R circle (centered on the torus axis) :

$$\oint_{\Gamma} \vec{B}_0 \cdot d\vec{l} = \mu_0 I_b$$



The magnetic field integration along the large radius R circle is proportional to the total current I_b that crosses the circle : This corresponds to the sum of the currents inside all torus coils. The magnetic magnitude is uniform along the toroidal circle (perimeter $2\pi R$) :

$$B_0 = \frac{\mu_0}{2\pi R} I_b$$

This toroidal magnetic field magnitude decreases with the large radius R .

1.2 Particle drifts through the magnetic field

In this configuration the magnetic field lines are not rectilinear (the curvature radius is the large radius R) and the magnetic field magnitude decreases with the large radius R :

$$B(R) = B(R_0) \frac{R_0}{R} .$$

This magnetic field configuration has an effect on the particle cyclotron motions. This effect is a trajectory guiding center motion drift across the magnetic field lines.

In order to simplify the equation forms, we will use a derivative expression reduced form for the time derivative and the spatial partial derivatives :

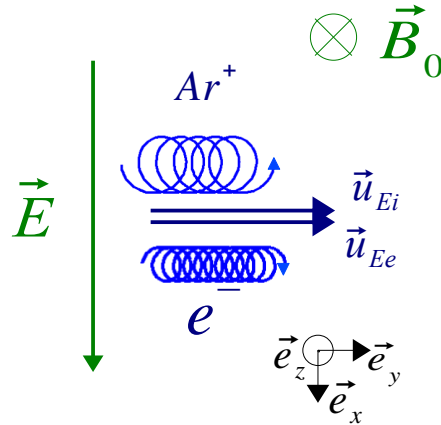
$$d_t = \frac{d}{dt} \quad \text{and} \quad \partial_x = \frac{\partial}{\partial x} .$$

2.a E cross B drift

Before looking at the magnetic field gradient and curvature drifts, we will study the E cross B drift. This first drift helps to analyze the more complex drifts linked with the magnetic field variations.

The presence of a uniform electric field \vec{E} perpendicular to the magnetic field \vec{B}_0 modifies the particle cyclotron motion inside a uniform magnetic field.

During a cyclotron cycle, the electric field accelerates then decelerates the particle along its trajectory. The particle velocity increases and then decreases. When the particle velocity is larger, the Larmor radius is also larger. The motion is not circular any more in the plane perpendicular to the magnetic field. The particle motion follows a cycloid. The cycloid drift direction is perpendicular to the electric and magnetic fields.



The particle motion inside a uniform magnetic field \vec{B}_0 and a uniform electric field \vec{E}_y perpendicular to \vec{B}_0 is described by the equation:

$$m_\alpha \vec{a}_\alpha = q_\alpha (\vec{E} + \vec{u}_\alpha \wedge \vec{B}_0) .$$

The particle free motion along the magnetic field direction is unchanged.

In the plane perpendicular to the magnetic field, the equations for both the acceleration components are:

$$a_{\alpha x} = \omega_{c\alpha} \left(u_{\alpha y} + \frac{E}{B_0} \right)$$

$$a_{\alpha y} = -\omega_{c\alpha} u_{\alpha x} .$$

We change the reference frame. It only affects the direction \vec{e}_y , perpendicular to

both the magnetic and the electric fields. The new reference frame $(u_{\alpha x}, u_{\alpha y}', u_{\alpha z})$ moves uniformly in relation to the initial reference frame :

$$u_{\alpha y}' = u_{\alpha y} + \frac{E}{B_0} .$$

The acceleration is unchanged : $a_{\alpha y}' = a_{\alpha y}$.

In the new reference frame, the motion equations are simplified :

$$\begin{aligned} a_{\alpha x} &= \omega_{c\alpha} u_{\alpha y}' \\ a_{\alpha y}' &= -\omega_{c\alpha} u_{\alpha x} . \end{aligned}$$

This corresponds to the cyclotron motion. The solution for the particle velocity in the new reference frame is :

$$\begin{aligned} u_{\alpha x}(t) &= u_{\perp\alpha} \cos[\omega_{c\alpha} t + \psi] \\ u_{\alpha y}'(t) &= -u_{\perp\alpha} \sin[\omega_{c\alpha} t + \psi] . \end{aligned}$$

Back in the initial reference frame, the particle velocity is :

$$\begin{aligned} u_{\alpha x}(t) &= u_{\perp\alpha} \cos[\omega_{c\alpha} t + \psi] \\ u_{\alpha y}(t) &= -u_{\perp\alpha} \sin[\omega_{c\alpha} t + \psi] - \frac{E}{B_0} . \end{aligned}$$

For the particle position, the solution is :

$$\begin{aligned} x_{\alpha}(t) &= x_{g\alpha} + \frac{u_{\perp\alpha}}{\omega_{c\alpha}} \sin[\omega_{c\alpha} t + \psi] \\ y_{\alpha}(t) &= y_{g\alpha 0} - \frac{E}{B_0} t + \frac{u_{\perp\alpha}}{\omega_{c\alpha}} \cos[\omega_{c\alpha} t + \psi] . \end{aligned}$$

The particle guiding center does not follow the magnetic field line but drifts across the magnetic field, with the velocity :

$$\vec{u}_{g\perp\alpha} = \vec{u}_{ExB} = -\frac{E}{B_0} \vec{e}_y = \frac{\vec{E} \wedge \vec{B}_0}{B_0^2} .$$

The drift velocity is known as the « E cross B » drift:

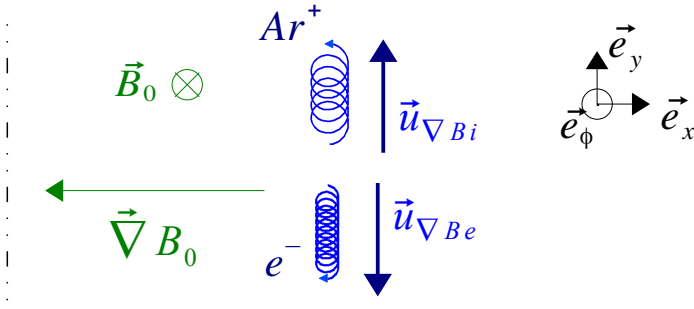
$$\vec{u}_{E\alpha} = \frac{\vec{E} \wedge \vec{B}_0}{B_0^2}$$

This drift is identical for all charged particles : the whole plasma drifts at this velocity.

2.b Particle drift due to the magnetic field gradient

We study the charged particle motion inside a non uniform magnetic field : the field magnitude increases in a direction perpendicular to the magnetic field lines.

The particle cyclotron motion inside such a magnetic field is perturbed. During a cyclotron motion cycle, the particle moves across zones where the magnetic field magnitude changes. Where the magnetic field is smaller, the particle Larmor radius is larger. The perturbed motion is a cycloid in the direction perpendicular to the magnetic field \vec{B}_0 and to its gradient $\vec{\nabla} B_0$.



The magnetic field magnitude gradient $\vec{\nabla} B_0$ is along \vec{e}_x .

We assume the magnetic field magnitude variations are small at the Larmor radius scale:

$$\frac{\nabla B_0}{B_0} \rho_{c\alpha} \ll 1 .$$

The perturbation is described using a 1st order Taylor polynomial development :

$$B_0(x) = B_0(x_0) \left[1 + \frac{\partial_x B_0(x_0)}{B_0(x_0)} (x - x_0) \right] .$$

We define β_{Bx} the inverse of the magnetic field gradient length L_{Bx} :

$$\beta_{Bx} = \frac{\partial_x B_0(x_{g\alpha})}{B_0(x_{g\alpha})} = \frac{1}{L_{Bx}} .$$

The cyclotron angular frequency varies along the same direction. We consider a perturbation development:

$$\omega_{c\alpha}(x_\alpha) = \omega_{c\alpha}(x_{g\alpha}) \left[1 + \beta_{Bx} (x_\alpha - x_{g\alpha}) \right] .$$

The motion equations inside the plane perpendicular to the magnetic field are modified:

$$\begin{aligned} a_{\alpha x} &= \omega_{c\alpha}(x_{g\alpha}) \left[1 + \beta_{Bx} (x_\alpha - x_{g\alpha}) \right] u_{\alpha y} \\ a_{\alpha y} &= -\omega_{c\alpha}(x_{g\alpha}) \left[1 + \beta_{Bx} (x_\alpha - x_{g\alpha}) \right] u_{\alpha x} . \end{aligned}$$

The 0th order solution corresponds to the cyclotron motion:

$$\begin{aligned} u_{\alpha x 0}(t) &= u_{\perp\alpha} \cos[\omega_{c\alpha} t + \psi] \\ u_{\alpha y 0}(t) &= -u_{\perp\alpha} \sin[\omega_{c\alpha} t + \psi] \\ x_{\alpha 0}(t) &= x_{g\alpha 0} + \rho_{c\alpha} \sin[\omega_{c\alpha} t + \psi] \\ y_{\alpha 0}(t) &= y_{g\alpha 0} + \rho_{c\alpha} \cos[\omega_{c\alpha} t + \psi] . \end{aligned}$$

The 1st order of the motion acceleration equation is :

$$\begin{aligned} a_{\alpha x 1} &= \omega_{c\alpha}(x_{g\alpha}) u_{\alpha y 1} + \omega_{c\alpha}(x_{g\alpha}) \left[1 + \beta_{Bx} (x_{\alpha 0} - x_{g\alpha 0}) \right] u_{\alpha y 0} \\ a_{\alpha y 1} &= -\omega_{c\alpha}(x_{g\alpha}) u_{\alpha x 1} - \omega_{c\alpha}(x_{g\alpha}) \left[1 + \beta_{Bx} (x_{\alpha 0} - x_{g\alpha 0}) \right] u_{\alpha x 0} . \end{aligned}$$

The 1st order equations depends on the 0th order solution:

$$\begin{aligned} a_{\alpha 1x} &= \omega_{c\alpha} u_{\alpha 1y} + \frac{1}{2} u_{\perp\alpha}^2 \beta_{Bx} [1 - \cos 2(\omega_{c\alpha} t - \varphi)] \\ a_{\alpha 1y} &= -\omega_{c\alpha} u_{\alpha 1x} + \frac{1}{2} u_{\perp\alpha}^2 \beta_{Bx} \sin 2(\omega_{c\alpha} t - \varphi) . \end{aligned}$$

The sinusoidal terms have a null mean value. These terms distort the cyclotron gyration but have no long term effect on the trajectory. We only keep the long term effect terms :

$$a_{\alpha 1x} = \omega_{c\alpha} u_{\alpha 1y} + \frac{1}{2} u_{\perp\alpha}^2 \beta_{Bx}$$

$$a_{\alpha 1y} = -\omega_{c\alpha} u_{\alpha 1x} .$$

By analogy with the motion equation system for the E cross B drift, the particle cyclotron motion is modified by a guiding center drift :

$$\vec{u}_{g\perp\alpha} = \frac{u_{\perp\alpha}^2}{2\omega_{c\alpha}} \frac{\partial_x B_0(x_{g\alpha})}{B_0(x_{g\alpha})} \vec{e}_y .$$

The drift is in the direction perpendicular to the magnetic field \vec{B} and its gradient $\vec{\nabla} B_0$. The vector expression of the particle magnetic gradient drift :

$$\vec{u}_{g\perp\alpha} = \frac{u_{\perp\alpha}^2}{2\omega_{c\alpha}} \frac{\vec{B}_0 \wedge \vec{\nabla} B_0}{B_0^2} .$$

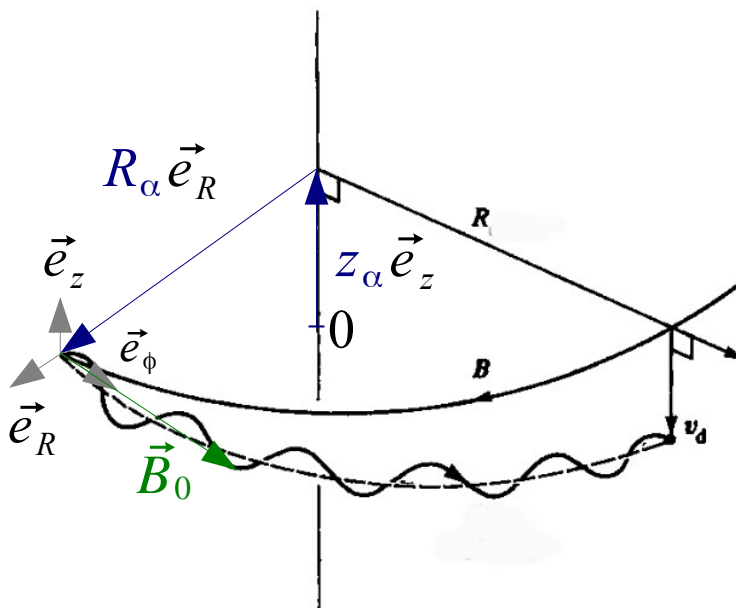
Taking into account the particle velocity distribution, the particle mean drift velocity :

$$u_{g\perp\alpha} \sim \frac{1}{2} \frac{\rho_{c\alpha}}{L_{Bx}} u_{th\alpha} .$$

Since the magnetic field gradient length (of the order of $1m$) is much smaller than the particle Larmor radius (of the order of $1mm$ for ions or $10\mu m$ for electrons), the drift velocity is much smaller than the particle thermal velocity.

2.c Particle drift due to the magnetic field curvature

The magnetic field curvature induces a charged particle drift across the magnetic field lines. Using the reference frame in rotation with the magnetic field line, the particle parallel velocity acts as a centrifugal force on the cyclotron motion. By analogy with the E cross B drift, this force induces a guiding center drift in the direction perpendicular to the magnetic field lines and to the magnetic field curvature direction: the drift is vertical.



The reference frame for this study is cylindrical $(\vec{e}_R, \vec{e}_\phi, \vec{e}_z)$. It rotates along the magnetic field line with the particle parallel velocity. The rotation center is on the magnetic field curvature axis (it corresponds to the torus axis). The azimuthal direction \vec{e}_ϕ is the magnetic field direction (the toroidal direction of the torus). The large radius R is magnetic field line curvature radius.

In the cylindrical reference frame, the particle velocity components are:

$$u_\phi = R d_t \phi$$

$$u_R = d_t R .$$

The partial derivatives of the reference frame unit vector are:

$$\partial_\phi \vec{e}_R = \vec{e}_\phi$$

$$\partial_\phi \vec{e}_\phi = -\vec{e}_R .$$

The unit vector temporal derivative along the particle trajectories are :

$$d_t \vec{e}_R = d_t \phi \partial_\phi \vec{e}_R = \frac{u_\phi}{R} \vec{e}_\phi$$

$$d_t \vec{e}_\phi = -\frac{u_\phi}{R} \vec{e}_R .$$

Since the reference frame rotates at the particle parallel velocity, the particle trajectory has only 2 components :

$$\vec{R}_\alpha(t) = R_\alpha \vec{e}_R(\phi_\alpha) + z_\alpha \vec{e}_z .$$

The particle velocity has 3 components :

$$\vec{u}_\alpha(t) = u_{R\alpha} \vec{e}_R + u_{\phi\alpha} \vec{e}_\phi + u_{z\alpha} \vec{e}_z .$$

We deduce the particle acceleration from the particle velocity derivative:

$$\vec{a}_\alpha(t) = d_t u_{R\alpha} \vec{e}_R + u_{R\alpha} d_t \vec{e}_R + d_t u_{\phi\alpha} \vec{e}_\phi + u_{\phi\alpha} d_t \vec{e}_\phi + d_t u_{z\alpha} \vec{e}_z .$$

We use the reference frame time derivative formulas:

$$\vec{a}_\alpha(t) = \left(d_t u_{R\alpha} - \frac{u_{\phi\alpha}^2}{R} \right) \vec{e}_R + \left(d_t u_{\phi\alpha} + \frac{u_{R\alpha} u_{\phi\alpha}}{R} \right) \vec{e}_\phi + d_t u_{z\alpha} \vec{e}_z .$$

The magnetic field is azimuthal :

$$\vec{B}_0 = B_0 \vec{e}_\phi .$$

The direction parallel to the magnetic field is the azimuthal direction :

$$\vec{e}_{\parallel} = \vec{e}_\phi .$$

The particle parallel velocity is the velocity azimuthal component :

$$\vec{u}_{\alpha\parallel} = u_{\phi\alpha} \vec{e}_\phi .$$

The particle motion equation in a magnetic field is:

$$m_\alpha \vec{a}_\alpha = q_\alpha \vec{u}_\alpha \wedge \vec{B}_0 .$$

Using the acceleration expression in the cylindrical reference frame :

$$d_t u_{R\alpha} - \frac{u_{\phi\alpha}^2}{R} = -\omega_{c\alpha} u_z$$

$$d_t u_{\phi\alpha} + \frac{u_{R\alpha} u_{\phi\alpha}}{R} = 0$$

$$d_t u_{z\alpha} = \omega_{c\alpha} u_R .$$

We assume the magnetic field curvature radius (the large radius) is large compare to the particle Larmor radius:

$$\rho_{c\alpha} \ll R .$$

We use a perturbation Taylor development. The small parameter is : $\frac{u_{\perp\alpha}}{R\omega_{c\alpha}}$.

The 0th order corresponds to the cyclotron motion :

$$d_t u_{R\alpha 0} = -\omega_{c\alpha} u_z$$

$$d_t u_{\phi\alpha 0} = 0$$

$$d_t u_{z\alpha 0} = \omega_{c\alpha} u_r .$$

The azimuthal velocity is the parallel velocity. This velocity is constant for the 0th order motion:

$$u_{\phi\alpha 0} = u_{\parallel\alpha 0} \text{ is constant ;}$$

$$u_{R\alpha 0} \text{ and } u_{z\alpha 0} \text{ are oscillating.}$$

The 1st order perturbation development is :

$$d_t u_{R\alpha 1} = \frac{u_{\phi\alpha 0}^2}{R} - \omega_{c\alpha} u_{z1}$$

$$d_t u_{\phi\alpha 1} = -\frac{u_{R\alpha 0} u_{\phi\alpha 0}}{R}$$

$$d_t u_{z\alpha 1} = \omega_{c\alpha} u_{r\alpha 1} .$$

The parallel 1st order velocity $u_{\phi\alpha 1} = u_{\parallel\alpha 1}$ oscillates because of the only oscillating factor $u_{R\alpha 0}$: its mean value is null over a cyclotron period or any integer number of cyclotron cycle:

$$\langle d_t u_{\phi\alpha 1} \rangle_T = 0 .$$

For the acceleration large radius component \vec{e}_R , the constant 0th order parallel velocity acts as a centrifugal velocity :

$$\vec{a} = \frac{u_{\phi\alpha 0}^2}{R} \vec{e}_R .$$

Both equations along \vec{e}_R and \vec{e}_z are analogous with the equations for the E cross B drift case. The centrifugal force replaces the electric force.

By analogy, the 1st order correction corresponds particle guiding center drift velocity :

$$\vec{u}_{g\alpha} = \frac{u_{\parallel\alpha}^2}{\omega_{c\alpha}} \frac{1}{R} \vec{e}_z .$$

The drift vector expression:

$$\vec{u}_{g\perp\alpha} = \frac{u_{\parallel\alpha}^2}{\omega_{c\alpha}} \frac{\vec{e}_R \wedge \vec{B}_0}{R B_0} .$$

Since the magnetic field curvature radius is large compared with the particle Larmor radius, The particle mean drift velocity is much smaller than the particle thermal velocity:

$$u_{g\perp\alpha} \sim \frac{\rho_{c\alpha}}{R} u_{th\alpha} \ll u_{th\alpha} .$$

1.3 Tore magnetized plasma particle drift

For a toric magnetic field produced by coils around the tore, both the magnetic field gradient and curvature are present.

Since the magnetic field magnitude decreases with the inverse of the large radius:

$$\vec{B}_0(R) = B_0 \frac{R_0}{R} \vec{e}_\phi .$$

The gradient is in the direction of the large radius :

$$\vec{\nabla} B_0 = -\frac{B_0}{R} \vec{e}_R .$$

The magnetic field curvature radius is also the large radius R .

The total drift velocity $\vec{u}_{torB\alpha}$ due to the magnetic field gradient and curvature is:

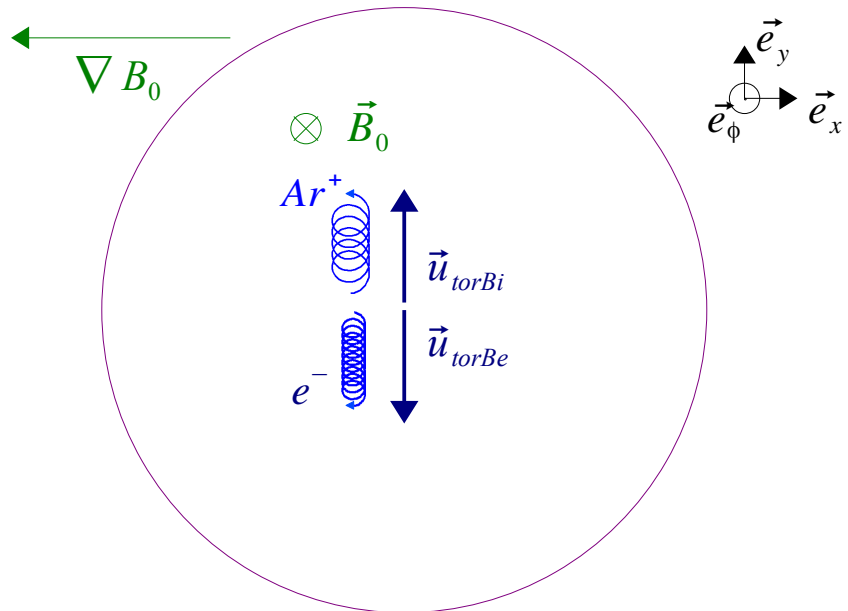
$$\vec{u}_{torB\alpha} = \frac{1}{\omega_{c\alpha}} \left(u_{\parallel\alpha}^2 + \frac{u_{\perp\alpha}^2}{2} \right) \frac{\vec{B}_0 \wedge \vec{\nabla} B_0}{B_0^2} .$$

The expression in the tore cylindrical reference frame:

$$\vec{u}_{torB\alpha} = \frac{1}{\omega_{c\alpha}} \left(u_{\parallel\alpha}^2 + \frac{u_{\perp\alpha}^2}{2} \right) \frac{\vec{e}_z}{R} .$$

3.a Charge separation

This magnetic field gradient and curvature drift is perpendicular to the magnetic field (The toroidal direction) and to the gradient direction $\vec{\nabla} B$ (the tore large radius direction) : the magnetic field gradient and curvature drift is vertical.

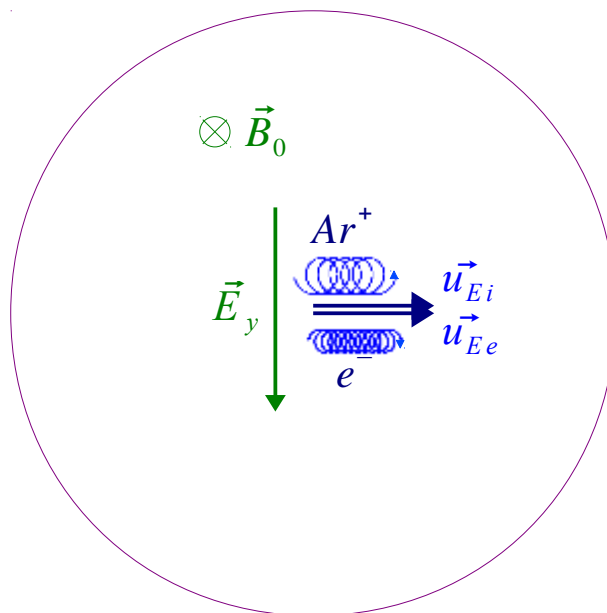


Since the particle angular frequency $\omega_{c\alpha}$ has the same sign as the particle charge, the ion drift \vec{u}_{torBi} and the electron drift \vec{u}_{torBe} are in opposite directions. The magnetic field gradient and curvature drifts for both kinds of charged particles are shown in the above poloidal section figure.

These opposite drifts tends to separate the opposite charges. The plasma is vertically polarized. This polarization creates a vertical electric field E_y . This vertical electric field prevents the particles from reaching the upper and lower walls.

3.b Secondary E cross B drift

The vertical electric field due to the plasma polarization by the magnetic field gradient and curvature drift creates a secondary E cross B drift.



This E cross B drift is perpendicular to the magnetic field direction (toroidal) and to the vertical electric field:

$$\vec{u}_{E\alpha} = \frac{\vec{E}_y \wedge \vec{B}_0}{B_0^2} .$$

This E cross B drift is the direction of the large radius.

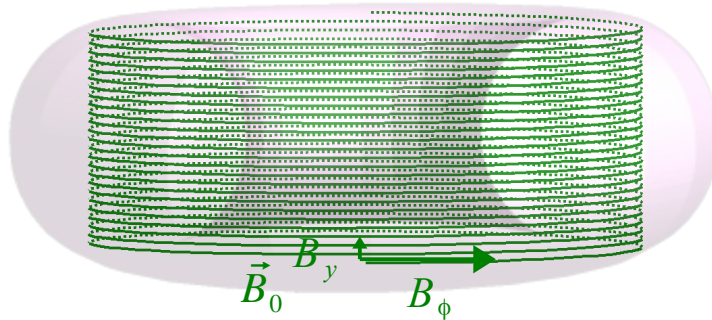
This secondary drift differs from the magnetic field drifts: this drift does not depend on the charge. The ion \vec{u}_{Ei} and electron \vec{u}_{Ee} drifts have the same magnitude and direction. The whole plasma moves along the large radius: The plasma tends to expand in the large radius direction.

This double drift effect tends to destabilize the plasma : there is no possible steady state plasma compatible with this magnetic field geometry.

3.c Stabilization with a vertical magnetic field component

In order to reduce this secondary drift due the charge separation created by the magnetic field drift, a small vertical magnetic field component B_y can be added to the main toroidal magnetic field $B_{\phi 0}$.

The magnetic field lines are not closed circles anymore. They form vertical helices with a small vertical step. Magnetic field lines are now opened. But 10 to 100 tore turns are needed to connect the upper wall to the lower wall along the magnetic field lines. The turn number depends on the ratio between the magnetic field vertical and toroidal components.



Since the particle motion is almost free along the magnetic field lines, the vertical electric field due to the charge separation will induce a charge motion along the magnetic field lines. This charge motion will reduce the vertical electric field: the electric potential tends to be homogenized vertically along the magnetic field lines. The secondary drift due to the electric field is reduced : the plasma is more stable.

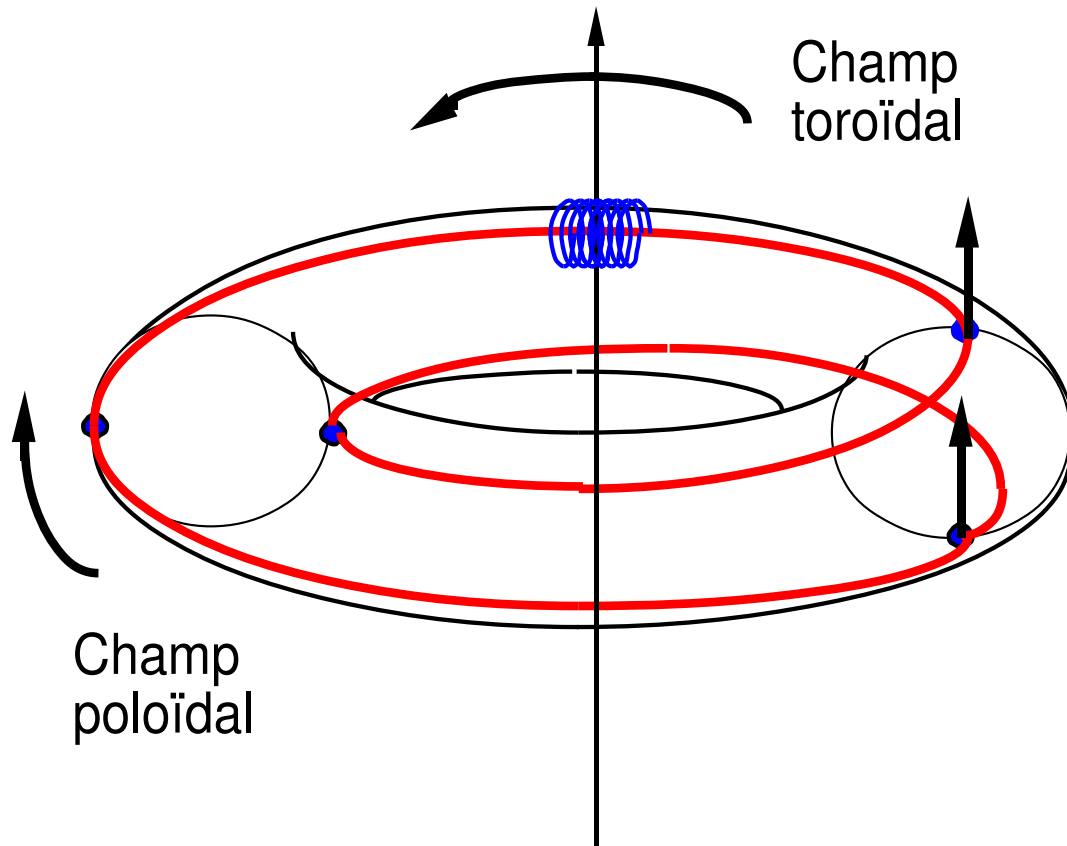
This stability enhancement has a cost : the magnetic field lines are now connected to the wall. The charged particle may reach the upper and lower wall after multiple tore turns following the magnetic field lines. The wall connection distance from the plasma center is of the order of a tens of meters.

3.d More complex magnetic configuration : the tokamaks and the stellarators

In order to obtain a stable plasma with closed magnetic field lines, we need a more complex magnetic configuration. The additional magnetic field component will be poloidal, and not vertical.

This additional poloidal magnetic field gives the magnetic field lines a helical shape, rotating in both the toroidal and the poloidal directions. Charged particles are guided by

these magnetic field lines. The vertical magnetic field gradient and curvature drift will guide them towards the plasma outside when the particles are above the torus median plane, but towards the plasma center when the particles are below: the distance from the particle to the center of the plasma oscillates at the frequency of the particle motion between the top and the bottom of the plasma.



This magnetic configuration can be obtained by inducing an electric current in the plasma, in the toroidal direction. This plasma toroidal current itself creates a poloidal magnetic field: this is the tokamak configuration.

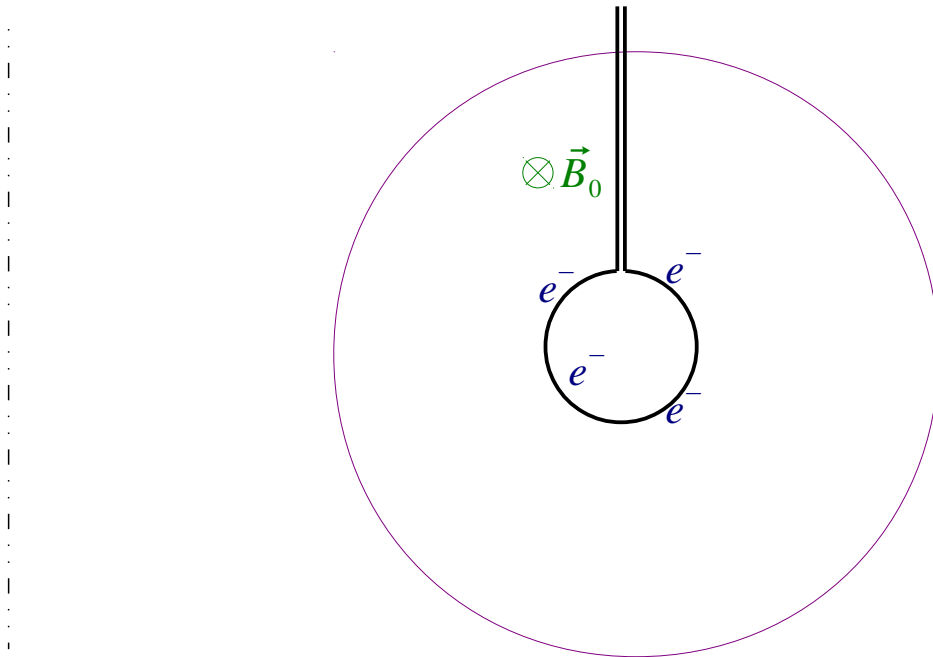
It can also be obtained by using coils with complex shapes, which directly create a magnetic field with a dominant toroidal component and a secondary poloidal component: this is the stellarator configuration.

2 Polarization by a filament

Plasma ionization

Inside Torix, the plasma ionization generally obtained thanks to a circular tungsten filament of 5 cm in diameter placed in the center of the poloidal section of the torus.

The filament temperature is of the order of a few thousand degrees. At this temperature, a large quantity of electrons in the filament have enough energy to extract themselves from the metal. The filament is put at a negative potential with respect to the tore wall: the electrons which leave the filament are attracted by the wall, but guided by the magnetic field. These electrons will be called primary electrons.



The primary electrons trajectory, guided by the magnetic field, is larger than electron mean free path inside neutral argon: part of the gas will be ionized. Each ionization will produce an electron and an ion. The electrons resulting from the neutral atom ionization will be qualified as the secondary electrons.

As the populations of secondary electrons and ions are produced by ionization, these 2 populations are equal: the primary electrons are supernumerary. The plasma around the filament has a negative potential.

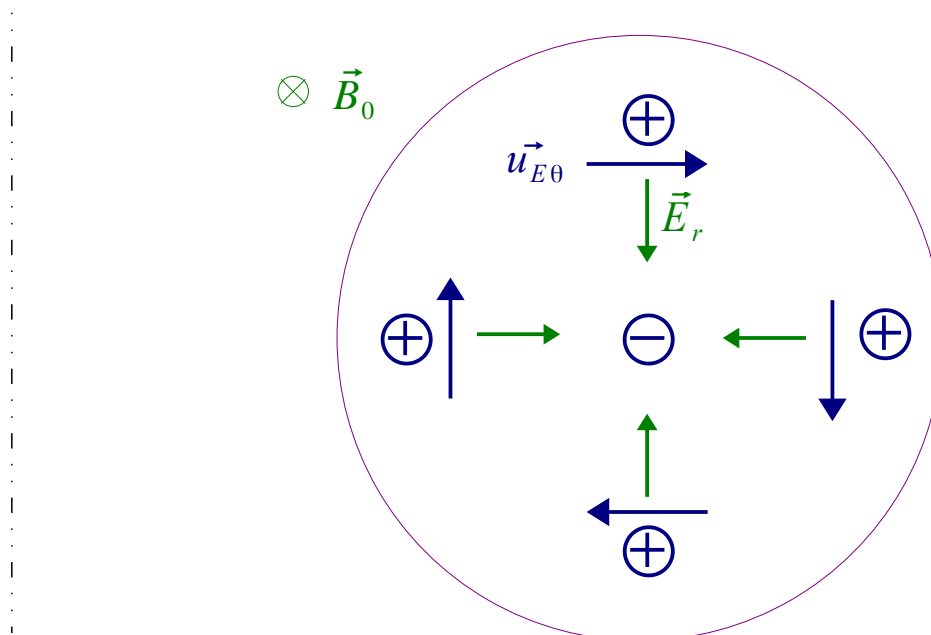
Plasma drift around the filament

Due to this filament negative potential, there is a small radius electric field \vec{E}_r centripetal around the filament. This electric field induces a drift in the poloidal direction around the filament at the velocity:

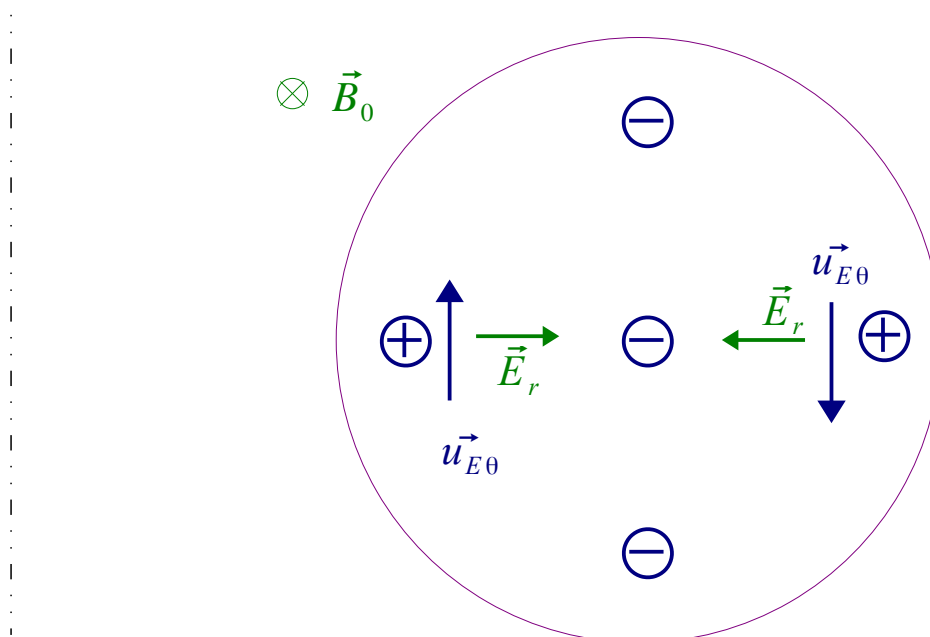
$$\vec{u}_{E\theta} = \frac{\vec{E}_r \wedge \vec{B}_0}{B_0^2} .$$

This filament polarization drift effect is superimposed on the magnetic field drift phenomena.

Since this drift velocity is the same for the ions and the electrons, the whole plasma rotates in the poloidal direction with this velocity.



With the addition of the vertical magnetic field component, the plasma potential tends to be homogenized along the vertical direction. Drift velocities in the poloidal direction are mainly present on the inner and outer edges:



The vertical magnetic field then has the effect of separating the inner and outer sides of the plasma.

Appendices

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