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Beyond scale separation in gyrokinetic turbulence

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Abstract

This paper presents the results obtained with a set of gyrokinetic codes based on a semi-Lagrangian scheme. Several physics issues are addressed, namely, the comparison between fluid and kinetic descriptions, the intermittent behaviour of flux driven turbulence and the role of large scale flows in toroidal ITG turbulence. The question of the initialization of full-F simulations is also discussed.

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1. Introduction

Predicting turbulent transport in nearly collisionless fusion plasmas requires solving the gyrokinetic equations for all species coupled to the Maxwell equations. In spite of considerable progress, several pending issues remain. In particular, the choice of the method for solving the Vlasov equation is subject to intense debate. On the one hand, Eulerian codes have proved their efficiency, but are potentially subject to numerical dissipation, therefore requiring high order numerical schemes (see, e.g. [1]). On the other hand, Lagrangian codes (typically particle in cell codes) benefit from widespread experience, but may be affected by numerical noise. The latter problem can be cured by techniques of ‘optimal loading’ [2] and filtering. This paper presents an alternative method based on a semi-Lagrangian scheme [3] applied to gyrokinetics [4, 5]. This technique allows one to compute the full distribution function with moderate dissipation and has been assessed in several ways. Here, some physics issues are presented, namely, the intermittency in flux driven systems, the comparison between kinetic and fluid descriptions (including an alternative description based on a water bag model, and the dynamics of flows in full torus simulations of ion temperature gradient (ITG) turbulence.

2. Flux driven turbulence

Intermittency is addressed here in a system whose fluid counterpart is known to exhibit intermittency when the system is flux driven, namely, 2D interchange turbulence in the scrape-off layer of tokamaks. The kinetic version of this system is described by the drift-kinetic equation

\[ \partial_t F + v_E \cdot \nabla F + v_D \cdot \nabla F = S, \]  

where \( F \) is a 3D distribution function, which depends on two spatial dimensions \( (x, y) \) (\( y \) is \( 2\pi \) periodical) and the energy \( E \). \( S \) is a source term that depends on energy and position, \( v_E \) is the \( E \times B \) electric drift velocity and \( v_D \) is the curvature drift velocity, which for this simplified system reads \( v_D = -E/eB_0\epsilon_0 \), where \( e \) is the ion charge, \( B_0 \) the magnetic field and \( \epsilon_0 \) a characteristic curvature length (the major radius in a tokamak). Self-consistency is ensured by coupling the Vlasov equation (1) to the Poisson equation

\[ \frac{n_{eq}}{T_0} (\phi - \langle \phi \rangle) - \nabla \cdot \left( \frac{n_{eq} m_i}{eB_0^2} \nabla \phi \right) = \int dEF - n_{eq}, \]  

where the bracket indicates an average over the periodic coordinate \( y \), \( \phi \) is the electric potential and \( n_{eq} \) is the
Where $S$ is extra to zero. The prescribed density and temperatures, while fluctuations are set up to an extra $E^{1/2}$ factor in expression (2) of the density [6]. The initial distribution function is a Maxwellian with turbulence driven. This has been done for a trapped ion turbulence (equations similar to equations (1) and (2), with an energy dependence on energy $E$). The source $S$ is such that the particle is located close to $x = 0$. The dependence on energy $(E/T - 1)$ is such that the particle source is null while heat is injected in the system [7]. We single out the evolution of the volume average energies of zonal flow and fluctuating potential (without zonal flows), and the central temperature. An example of the rich dynamics of these three fields is reported in figure 1. When a large increase in the magnitude of the zonal flows occurs, the fluctuations are quenched and the core temperature builds up while the edge temperature decreases, in agreement with the usual picture for a transport barrier. In the first stage, the increase in the temperature gradient through the system takes place at the same time as the magnitude of the zonal flows decays away until an abrupt relaxation event takes place.

The GYSELA code is usually run without sources. In this case boundary conditions are set such that the distribution function matches a Maxwellian with fixed density and temperature at both radial ends of the simulation box. However tokamak plasmas are flux driven, i.e. the temperature is controlled by a heat source and not by fixed values at the edges. Hence it is quite crucial to study systems where turbulence is flux driven. This has been done for a trapped ion turbulence (equations similar to equations (1) and (2), with an extra $E^{1/2}$ factor in the energy integration [6]) by implementing an energy dependent source term in the Vlasov equation (1). The source $S$ is chosen of the form $S_0(x)(E/T - 1)$, where $S_0(x)$ is a Gaussian located close to $x = 0$. The dependence on energy $(E/T - 1)$ is such that the particle source is null while heat is injected in the system [7]. We single out the evolution of the volume average energies of zonal flow and fluctuating potential (without zonal flows), and the central temperature. An example of the rich dynamics of these three fields is reported in figure 1. When a large increase in the magnitude of the zonal flows occurs, the fluctuations are quenched and the core temperature builds up while the edge temperature decreases, in agreement with the usual picture for a transport barrier. In the first stage, the increase in the temperature gradient through the system takes place at the same time as the magnitude of the zonal flows decays away until an abrupt relaxation event takes place.

In the second stage, the relaxation governs a significant turbulence mixing during which the zonal flows are reduced, the radial temperature gradient flattens out and a large energy flux is transferred to the edge. The zonal flows build up again, then starting over such a predator–prey cycle. In such simulations the interplay between the zonal flows and the turbulent drift transport generates two distinct behaviours. In some cases there is a strong zonal flow overshoot that generates a transient transport barrier, while in other cases the zonal flows govern a saturation of the turbulent transport. This investigation of flux driven transport in a kinetic simulation confirms previous findings with fluid simulations for edge turbulence [8]. In particular the probability distribution function of temperature fluctuations is non-Gaussian (figure 2). Also it is found that a reduced description of the velocity space does not capture the right physics of an interchange turbulence described by equations (1) and (2). This motivates a detailed comparison between fluid and kinetic descriptions of interchange turbulence, presented in the next section.

3. Reduced 3D simulations: kinetic versus fluid descriptions

The fluid equations for interchange turbulence are obtained by computing the two first moments of equation (1), i.e.

$$\frac{\partial}{\partial t}N + v_E \cdot \nabla N + v_D \cdot \nabla P = 0,$$

$$\frac{\partial}{\partial t}P + v_E \cdot \nabla P + v_D \cdot \nabla Q = 0,$$

where $N$ is the density, $P$ the pressure and $Q = \int_0^{\infty} dEE^2F$ is the heat flux. For kinetic and fluid descriptions to match, an expression for $Q$, i.e. a closure, must be chosen. The simplest one consists of setting $Q = \Gamma PT$, where $\Gamma$ is the adiabatic index (for a 2D system, a Maxwellian yields $\Gamma = 2$). To minimize the numerical errors, the same semi-Lagrangian scheme has been used to solve both the kinetic and the fluid equations. For the fluid case, this is done by solving the Vlasov equation for two energies $E_{\pm}$:

$$\frac{\partial}{\partial t}F_{\pm} + v_E \cdot \nabla F_{\pm} + v_D \cdot \nabla F_{\pm} = 0,$$
and (4) with diffusion added to match the instability threshold. The comparison is done with and without zonal flows.

\[ \text{3D kinetic interchange turbulence equation (1) to the fluid equations (3) and (4) differs from the kinetic description, using the conservation property of the distribution function in the phase space. Accordingly, a discrete distribution function is assumed along the velocity direction, taking the form of a multi-step-like function. The water bag model provides a bridge between kinetic and fluid descriptions: the water bag model is far from a Maxwellian. Nevertheless, as this is a rather new approach, it is possible that non-diffusive closure schemes, which have proved their efficiency in the past for ad hoc closures [11, 12], would work [13]. An alternative solution, based on a water bag representation, is currently investigated and is presented in the next section.}

\[ \begin{align*}
\text{Figure 3. Heat flux versus distance to the threshold } \kappa, \text{ comparing a 3D kinetic interchange turbulence equation (1) to the fluid equations (3) and (4) with diffusion added to match the instability threshold. The comparison is done with and without zonal flows.}
\end{align*} \]

\[ \begin{align*}
\text{Figure 4. Difference between the distribution function and the initial Maxwellian distribution for the kinetic interchange case.}
\end{align*} \]
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Figure 5. Instability diagram in the plane \( \omega r^* = f(\omega n^*) \) for a multi-water bag distribution function and a Maxwellian.

Maxwellian background distribution function. Linearizing the set of equations (10), one gets the local dispersion relation of a drift-kinetic multi-water bag plasma for equal ion and electron temperatures, and hydrogenic species:

\[
\varepsilon(\omega) = 1 - \sum_{j=1}^{N} \alpha_j \frac{k_0^2 c_s^2 - \omega^2}{k_0^2 c_s^2 - \omega^2} = 0, \tag{11}
\]

where \( \alpha_j \) is the initial (‘equilibrium’) parallel velocity \( v_{ij} \), \( c_s = (T_{eq}/m_i)^{1/2} \) is the thermal velocity, \( \alpha_j = n_{eq,j}/n_{eq} \) is related to the density of the \( j \)th bag \( n_{eq,j} = 2\alpha_j A_j \) and the diamagnetic frequency for each bag is defined as

\[
\omega^*_j = \frac{k_0 T_{eq}}{eB} \partial_{n_{eq,j}} n_{eq,j}. \tag{12}
\]

Finding the complex roots of equation (11) provides the linear growth rate, whose existence and value essentially depend on the values of the density and temperature diamagnetic frequencies \( \omega^*_n = k_0 T_{eq} \partial_{n_{eq}} n_{eq} / eB \) and \( \omega^*_T = k_0 T_{eq} \partial_{T_{eq}} (n_{eq} T_{eq}) / eB \). The one bag case can be shown to be equivalent to the adiabatic fluid description, without any instability, and a constant ratio between temperature and density gradients (\( L_n / L_T = 2 \)). The instability threshold for ITG instability is found to be close to the results obtained from continuous Maxwellian distribution function when \( N = 10 \), except for large density gradients (see figure 5) [14]. The method remains to be tested in the non linear regime, first for the interchange turbulence described in section 2, second in the 4D slab geometry where it will be compared with a 4D version of the GYSELA code [5,15] and finally to the full 5D toroidal case.

5. Global non-perturbative simulations of toroidal ITG turbulence

The GYSELA code has been upgraded to run 5D simulations of toroidal ITG turbulence [16]. The codes solves the gyrokinetic equation

\[
\partial_t F + v_E \cdot \nabla F + v_D \cdot \nabla F + v_\perp \nabla_\perp F + v_\parallel \partial_{v_\parallel} F = 0, \tag{13}
\]

where

\[
v_E = B \times \nabla (J \cdot \phi); \quad v_D = \frac{m_i v_\perp^2}{eB} \left( \frac{B}{B} \times \frac{N}{R} \right) + \frac{\mu}{e} \left( \frac{B}{B} \times \nabla B \right), \tag{14}
\]

\[
m_1 \psi = -\left[ \frac{B}{B} + \frac{m_1 \psi}{eB} \left( \frac{B}{B} \times \frac{N}{R} \right) \right] \cdot (\mu \nabla B + e \nabla J \cdot \phi), \tag{15}
\]

where \( N/R \) is the field line curvature. The Poisson equation reads

\[
\frac{n_{eq} e}{T_0} (\phi - \langle \phi \rangle) - \nabla_\perp \cdot \left( \frac{n_{eq} m_1}{eB} \nabla_\perp \phi \right) = \int B d\mu \psi J \cdot F - n_{eq}, \tag{16}
\]

where \( J \) is the gyroaverage operator (multiplication by \( J_0(k_\perp \rho_c) \) in Fourier space, \( \rho_c \) is the gyroradius—the present implementation of this operator is based on a Padé representation of the Bessel function \( J_0 \)). The code uses a simplified geometry based on a set of circular centred magnetic surfaces with a magnetic field \( B = B_0 R / (r + r/q R \rho_c) \) and \( R = R_0 + r \cos(\theta) \). No source is added in this version of the code. Boundary conditions are such that the distribution function matches a Maxwellian with fixed density and temperature at both radial ends of the simulation domain.
The parameters of the simulation correspond to the Cyclone base case [9] with a gyroradius normalized to the plasma size \( \rho_s \) equal to 0.005.

Several numerical tests have been done to check the accuracy when solving the Vlasov equation, in particular regarding the conservation of motion invariants [16]. Also the linear growth rates agree well with the expected values, and the \( n = 0, m = 0 \) component of the electric potential, labelled \( \phi_{00} \), evolves as expected. Namely, it decays while emitting geodesic acoustic modes, with a damping rate that agrees with the value calculated by Sugama and Watanabe [17].

The flow ultimately relaxes towards a non-zero residual value, as prescribed by Rosenbluth and Hinton [18]. In the following stages, the system develops in a second stage, but the shear flow is too strong to allow a growth of turbulence [19, 20]. In some cases, \( \phi_{00} \) is self-consistently calculated, a standard shearless ITG turbulence develops, as shown in figure 7.

For an initial distribution function is a Maxwellian of the form

\[
F_{eq}(r, E) = \frac{n_{eq}(r)}{\left(2\pi m T_{eq}(r)\right)^{3/2}} \exp\left\{-\frac{E}{T_{eq}(r)}\right\},
\]

where \( E = m v_i^2/2 + \mu B \), and in the case where \( \phi_{00} \) is artificially suppressed, a standard shearless ITG turbulence develops, as shown in figure 7.

However when \( \phi_{00} \) is self-consistently calculated, large scale steady flows appear, which prevent the onset of turbulence (see figure 8). This is a consequence of the equilibrium distribution function not being a function of the motion invariants. If the distribution function is a function of the minor radius instead of the canonical toroidal moment, no parallel flow exists initially to balance the charge separation associated with the ion curvature drift. Hence a polarization drift appears to ensure the charge balance. Parallel flows develop in a second stage, but the shear flow is too strong to allow a growth of turbulence [19, 20]. In some cases, turbulence ultimately develops but on a slow time scale. It is in fact much safer to ensure the charge balance by allowing the development of parallel flows, in particular \( n = 0, m = 1 \) components (Pfirsch–Schlüter effect).

A similar problem was already mentioned by Idomura et al. [21] and by Angelino et al. [22]. It is cured by prescribing an equilibrium distribution function that is canonical, i.e. a function of motion invariants. The choice here is the same as the one proposed in the [22], that is a distribution function of form (17), with the minor radius \( r \) replaced by the motion invariant \( \tilde{r} \) defined as

\[
\tilde{r} = r_0 - \frac{q_0}{q} - \frac{q_0}{q} \int_{r_0}^{r_0} \frac{rdr}{q} + \frac{q_0}{q} \frac{m}{\mu B_0} \left( R v_r - R_0 \tilde{v}_r \right),
\]

where \( \tilde{v}_r = \text{sign}(v_r) \left[ \frac{2}{m} (E - \mu B_0) \right]^{1/2} \) and \( \Theta \) is a step function, \( B_m \) is the maximum of magnetic field in the whole box (all quantities with a label 0 are defined at half-radius of the simulation box), and \( q \) is the average parallel velocity and electric potential evolve towards steady values. The final state is turbulent, and the level of fluctuations \( e(\phi - \phi_{eq})/T_{eq} \) lies in the expected range (figure 9) (\( \phi_{eq} \) is the time average of \( \phi_{00} \)). In this configuration the corresponding \( E \times B \) shear rate is small enough not to affect the turbulent transport. When comparing figures 7 and 9, it appears that the radial size of vortices is reduced by the shearing effect of zonal flows, as expected [23]. Also the thermal flux is in the right order of magnitude when compared with the Cyclone curve giving the flux versus the normalized temperature gradient (figure 10) [9]. The simulation shown in
Developing equation (20) at first order in \( \rho_\ast \) and calculating the 2nd moment yield the fluid velocity

\[
V = W_{eq}(\psi) \frac{B}{B_{eq}^\ast(\psi)} \left( \frac{d\phi_{eq}}{d\psi} + \frac{1}{n_{eq}} \frac{dp_{eq}}{d\psi} \right) R^2 \nabla \psi.
\]  

(22)

This expression is fully consistent with the usual prescription for the equilibrium velocity in a tokamak and satisfies the force balance equation \( E + V \times B - \nabla \mu/n e = 0 \). In a collisionless code, the choice of the function \( W_{eq} \) is arbitrary. The choice equation (20) minimizes the mean parallel velocity, as mentioned in [22]. Adding collisions would lead to a different result as the neoclassical viscous damping prescribes a value of \( W_{eq} \) that usually corresponds to a finite parallel velocity. Hence the choice of the initial distribution function is intimately related to the structure of the initial mean flow, which can be chosen by prescribing the values of \( \phi_{eq} \) and \( W_{eq} \).

6. Conclusion

A sequence of gyrokinetic codes with increasing dimensionality and based on a semi-Lagrangian scheme has been developed and used to clarify several physics issues related to the interplay between flows and turbulence and the scale separation assumption. Flux driven interchange turbulence has been simulated. As in previous fluid flux driven simulations, strong intermittency is observed. It is related to abrupt relaxations of the temperature profile, correlated with fast events in the zonal flow and fluctuation dynamics. Also turbulent fluxes for a 3D interchange turbulence are found to be different when using fluid or kinetic equations. This discrepancy is partially due to the different behaviour of zonal flows. Moreover the shape of the distribution function in the energy space is found to differ significantly from a Maxwellian, suggesting that part of the difference is due to resonant wave/particle interaction. An alternative to fluid description that describes kinetic effects, the water bag representation, has been studied. It is shown to reproduce correctly the kinetic linear growth rate and frequencies with a limited amount of bags. It remains, however, to be tested in the non-linear regime. Finally a 5D version of the GYSELA code has been developed to study toroidal ITG turbulence. The linear growth rates agree with the values expected for the Cyclone base case and zonal flows behave as expected. The choice of the initial distribution function is
crucial for the non-linear stage. The best option is to choose a function of motion invariants to avoid the development of spurious large scale flows, which may prevent the development of turbulence. Once the distribution function is properly initialized, the simulations produce turbulent heat fluxes in the expected range.

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