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CLUSTER SOFTWARE TOOLS

PART I

COORDINATE TRANSFORMATIONS LIBRARY

par

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EUROPEAN SPACE AGENCY

Study of the Cluster Mission
Planning Related Aspects
within the Numerical Simulations Network

CLUSTER SOFTWARE TOOLS

PART I

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Version 1.1

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CONTENTS

GENERAL REMARKS

This document describes a coordinate transformation library (ROCOTLIB), especially developed in the framework of the CLUSTER mission. Most of the frame of reference used are in geocentric system, and are thus independent of the position of the point of observation. Nevertheless, local frames are also considered. On the other hand, many useful subroutines allowing calendar conversions are also delivered, because time is often given in various format, such as decimal Julian day, or month-day-year, hour-min-sec and so on.

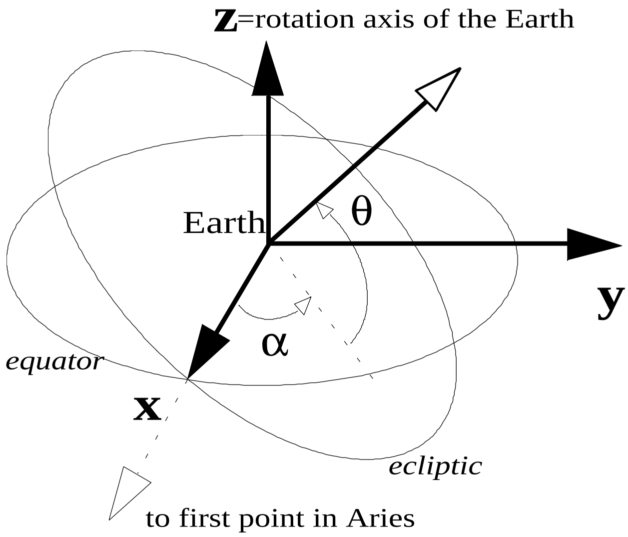
In part I, we recall the definition of the different geophysical coordinate systems used. In part II, a list of transformations is given. Part III gives mathematical expressions of corresponding transformation matrix. Part IV is dedicated to calendar conversion problems. Part V describes the ROCOTLIB developed from the preceding formula, where input/output of each subroutine are detailed. At least, part VI describes directions for use, with examples of Fortran user program. A standard test program, for checking the good installation of the software, is also given with the correct results.

This library is written in FORTRAN 77 language, and can be supported by any computer where this compiler is available.

I- DESCRIPTION OF COORDINATE SYSTEMS

Most of the coordinate systems described are geocentric, with the exception of the dipole meridian system and the VDH system, which are local coordinate systems and thus depend of the position of the point of observation.

1) Geocentric Equatorial Inertial system (GEI)



The Z-axis is parallel to the rotation axis of the Earth.
The X-axis is defined by the intersection of the equator plane and the ecliptic plane, and is pointing towards the first point of Aries (Sun position at the vernal equinox).

one can define the *right ascension* α and the *declination* θ as:

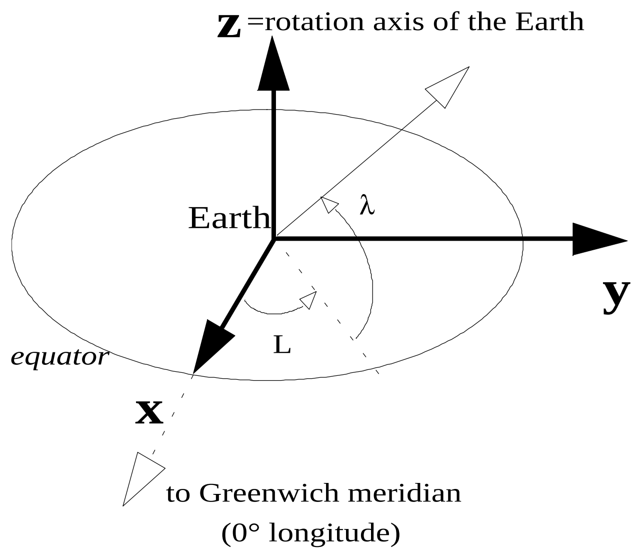
right ascension $\alpha = \tan^{-1}(V_y/V_x)$

with α in [0°,180°] for $V_y>0$
α in [180°,360°] for $V_y<0$

declination $\theta = \sin^{-1}(V_z/V)$

with θ in [-90°,90°]

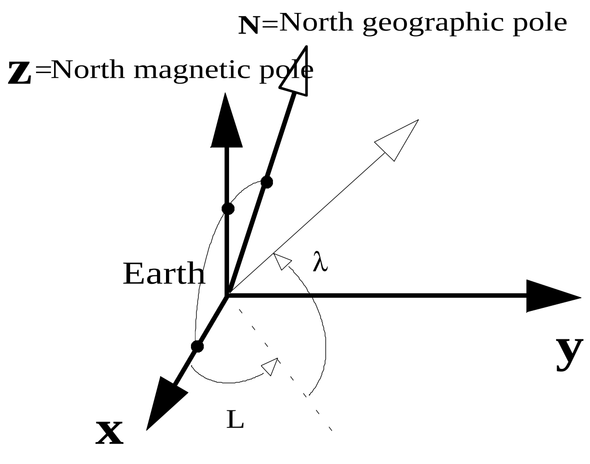
2) Geographic system (GEO)



The Z-axis is parallel to the rotation axis of the Earth.
The X and Y axis are included in the equator plane.
The X axis is pointing from the centre of the Earth to the Greenwich meridian (0° longitude).

The GEO system is fixed with the rotating Earth. Longitude L and latitude λ are defined in this system in the same way as right ascension and declination in GEI system.

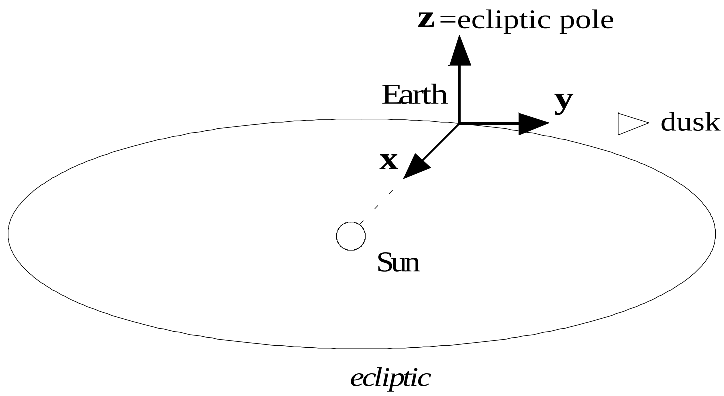
3) Geomagnetic system (MAG)



The Z-axis is parallel to the magnetic dipole axis.
If N is the North geographic pole, the N,Z and X vector are in the same plane.
The Y-axis is defined as Y=-Z x N

The MAG system is fixed with the rotating Earth. The magnetic longitude L and magnetic latitude λ are defined in this system in the same way as right ascension and declination in GEI system.

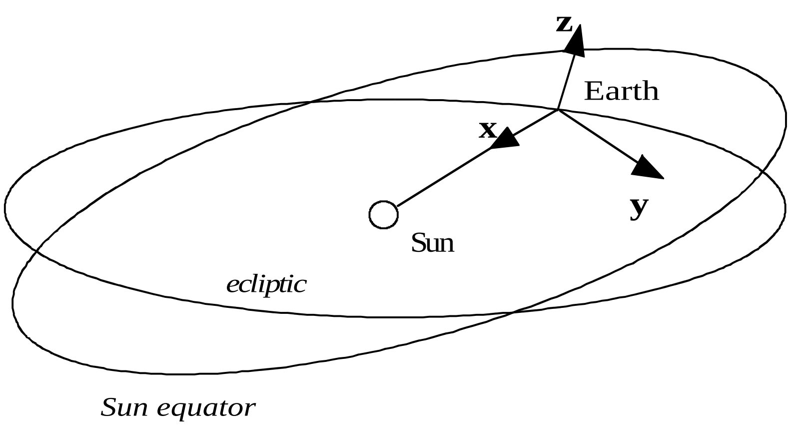
4) Geocentric Solar Ecliptic system (GSE)



The X-axis is pointing from the Earth towards the Sun.
The X-axis and the Y-axis are include in the ecliptic plane.
The Y-axis is pointing toward the dusk, opposing to the planetary motion.

The Z-axis is parallel to the ecliptic pole. The GSE system has a yearly rotation with respect to the inertial system.

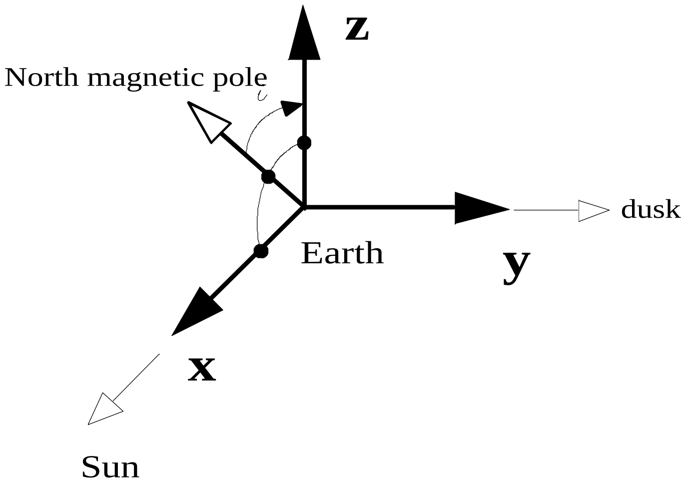
5) Geocentric Solar Equatorial system (GSEQ)



The X-axis is pointing toward the Sun and include in the ecliptic plane.
The Y-axis is parallel to the Sun's equatorial plane (inclined to the ecliptic)

X-axis is not necessarily in the Sun's equatorial plane;
Z-axis is not necessarily be parallel to the Sun's axis of rotation (which is perpendicular to Y, and thus in the X-Z plane);
Z-axis is chosen to be in the same sense as the ecliptic pole, i.e. northwards.

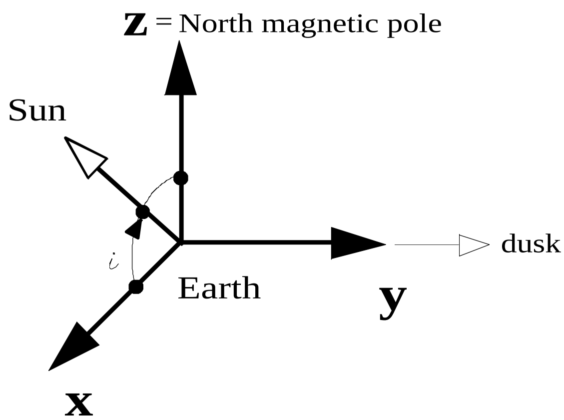
6) Geocentric Solar Magnetospheric system (GSM)



The X-axis is pointing from the Earth towards the Sun.
The X-Z plane contains the dipole axis.
The Y-axis is perpendicular to the Earth's magnetic dipole, towards the dusk and include in the magnetic equator plane.

The positive Z-axis is chosen to be in the same sense as the northern magnetic pole; the dipole tilt angle i is positive when the north magnetic pole is tilted towards the Sun. In addition to a yearly period due to the motion of the Earth about the Sun, the GSM system rocks about the Solar direction with a 24 h period.

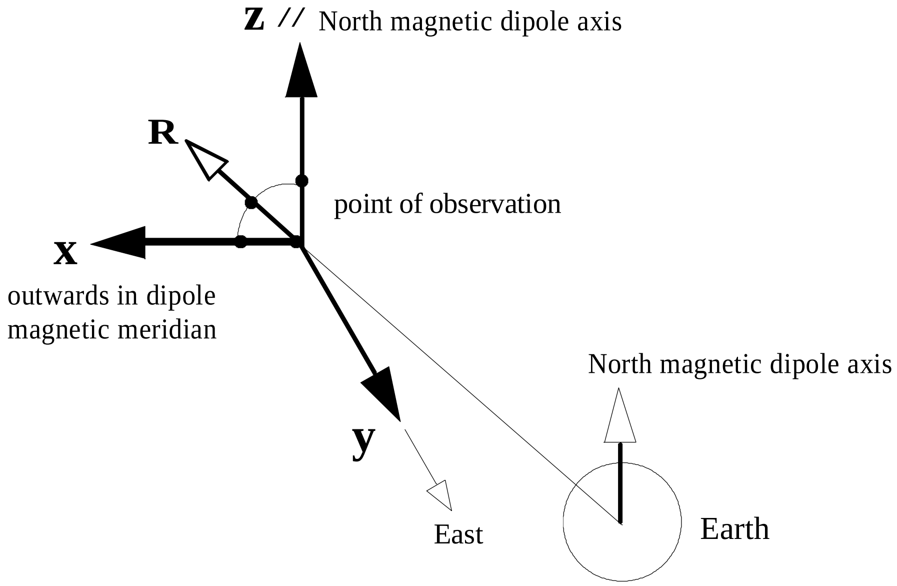
7) Solar Magnetic system (SM)



The Z-axis is parallel to the North magnetic dipole.
The X-Z plane contains the direction of the Sun.
The Y-axis is perpendicular to the Earth-Sun line toward dusk.

The SM system rotates with both a yearly and a daily period with respect to the inertial system.

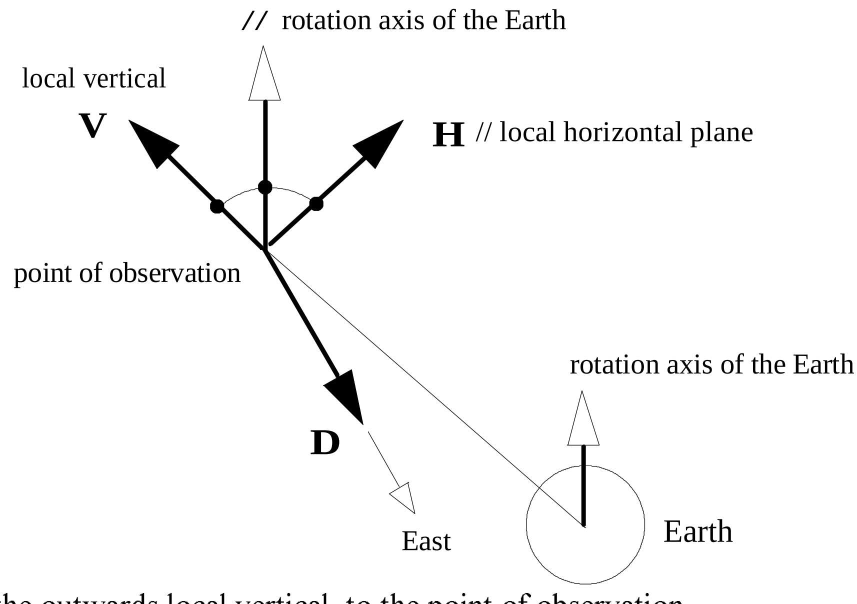
8) Dipole Meridian system (DM)



The Z-axis is parallel to the North magnetic dipole axis.
The X-Z plane contains the direction **R** of the point of observation, from the Earth, and is a dipole magnetic meridian plane.
The Y-axis is perpendicular to the **R** vector, eastwards.

This system is a local coordinate system, which is dependent of the position of the point of observation from the Earth.

9) Vertical Dusk Horizontal system (VDH)



The V-axis is the outwards local vertical, to the point of observation.
The H-axis is parallel to the horizontal local plane, positive to the North.
The V-H plane is a geographic meridian plane. The D-axis is azimuthal, eastwards.

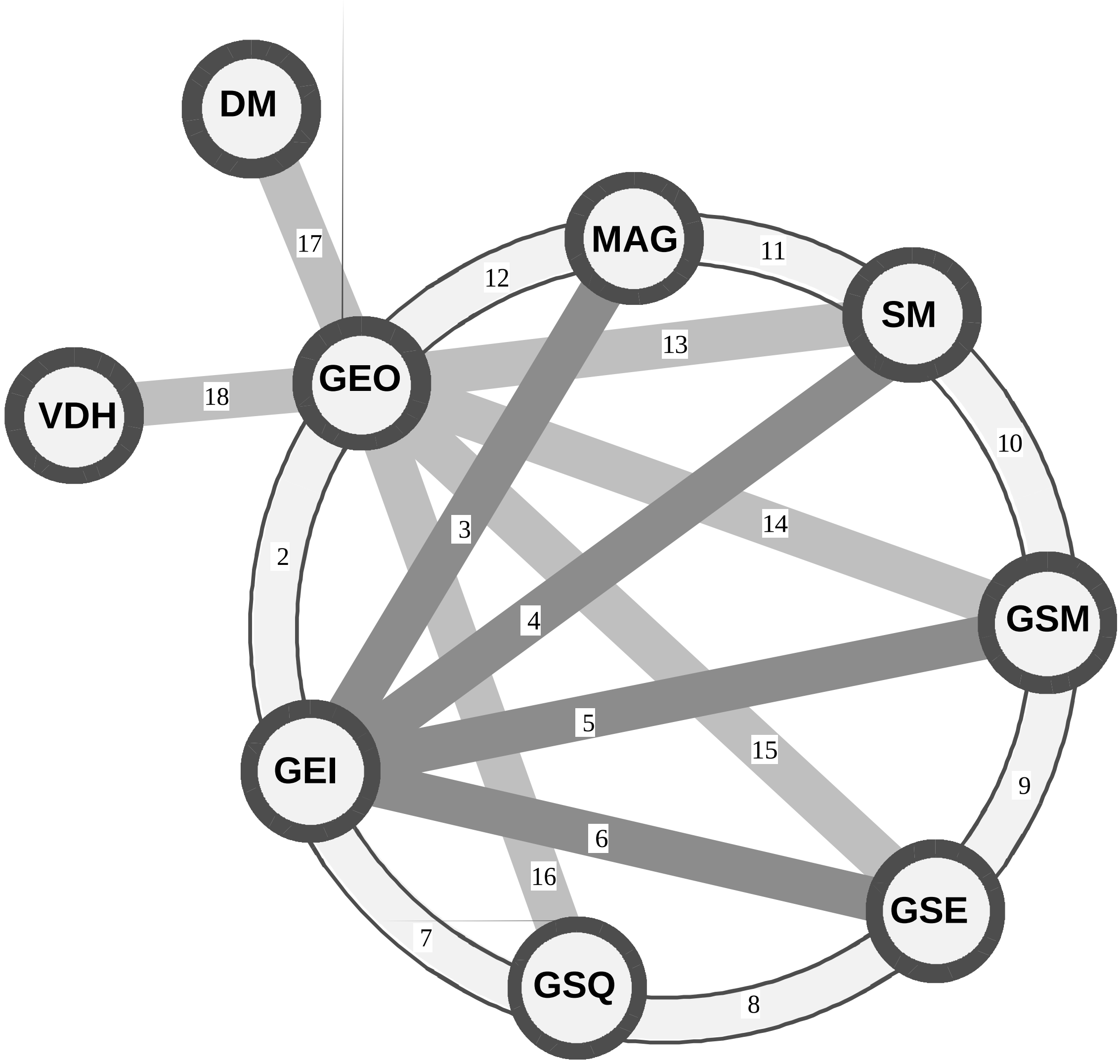
As DM system, this system is a local coordinate system, which is dependent of the position of the point of observation from the Earth.

II- DIAGRAM OF TRANSFORMATIONS

1) general remarks

Among all coordinates systems described in section I, we have single out two particular systems more frequently encountered: the Geographic system (GEO) and the Geocentric Equatorial Inertial system (GEI). From these two system, we have computed all transformations to directly convert theses coordinates into any else other. Furthermore, other direct "circular" transformations between the other system are also given, as explained on the schematic diagram below.

2) schematic diagram of transformations



3) list of transformations

All different coordinate transformations are listed below; number correspond to § number of section III, and are mentioned in the above schematic diagram.

	name	*input coordinates*		*output coordinates*
2)	geigeo	Geocentric Equatorial Inertial (GEI)	-->	Geographical (GEO)
	geogei	Geographical (GEO)	-->	Geocentric Equatorial Inertial (GEI)
3)	geimag	Geocentric Equatorial Inertial (GEI)	-->	Magnetic dipole (MAG)
	maggei	Magnetic dipole (MAG)	-->	Geocentric Equatorial Inertial (GEI)
4)	geism	Geocentric Equatorial Inertial (GEI)	-->	Solar Magnetic (SM)
	smgei	Solar Magnetic (SM)	-->	Geocentric Equatorial Inertial (GEI)
5)	geigsm	Geocentric Equatorial Inertial (GEI)	-->	Geocentric Solar Magnetospheric (GSM)
	gsmgei	Geocentric Solar Magnetospheric (GSM)	-->	Geocentric Equatorial Inertial (GEI)
6)	geigse	Geocentric Equatorial Inertial (GEI)	-->	Geocentric Solar Ecliptic (GSE)
	gsegei	Geocentric Solar Ecliptic (GSE)	-->	Geocentric Equatorial Inertial (GEI)
7)	geigsq	Geocentric Equatorial Inertial (GEI)	-->	Geocentric Solar Equatorial (GSQ)
	gsqgei	Geocentric Solar Equatorial (GSQ)	-->	Geocentric Equatorial Inertial (GEI)
8)	gsegsq	Geocentric Solar Ecliptic (GSE)	-->	Geocentric Solar Equatorial (GSQ)
	gsqgse	Geocentric Solar Equatorial (GSQ)	-->	Geocentric Solar Ecliptic (GSE)
9)	gsegsm	Geocentric Solar Ecliptic (GSE)	-->	Geocentric Solar Magnetospheric (GSM)
	gsmgse	Geocentric Solar Magnetospheric (GSM)	-->	Geocentric Solar Ecliptic (GSE)
10)	gsmsm	Geocentric Solar Magnetospheric (GSM)	-->	Solar Magnetic (SM)
	smgsm	Solar Magnetic (SM)	-->	Geocentric Solar Magnetospheric (GSM)
11)	smmag	Solar Magnetic (SM)	-->	Magnetic dipole (MAG)
	magsm	Magnetic dipole (MAG)	-->	Solar Magnetic (SM)
12)	geomag	Geographical (GEO)	-->	Magnetic dipole (MAG)
	maggeo	Magnetic dipole (MAG)	-->	Geographical (GEO)
13)	geosm	Geographical (GEO)	-->	Solar Magnetic (SM)
	smgeo	Solar Magnetic (SM)	-->	Geographical (GEO)
14)	geogsm	Geographical (GEO)	-->	Geocentric Solar Magnetospheric (GSM)
	gsmgeo	Geocentric Solar Magnetospheric (GSM)	-->	Geographical (GEO)
15)	geogse	Geographical (GEO)	-->	Geocentric Solar Ecliptic (GSE)
	gsegeo	Geocentric Solar Ecliptic (GSE)	-->	Geographical (GEO)
16)	geogsq	Geographical (GEO)	-->	Geocentric Solar Equatorial (GSQ)
	gsqgeo	Geocentric Solar Equatorial (GSQ)	-->	Geographical (GEO)
17)	geodm	Geographical (GEO)	-->	Dipole Meridian (DM)
	dmgeo	Dipole Meridian (DM)	-->	Geographical (GEO)
18)	geovdh	Geographical (GEO)	-->	Vertical Dusk Horizontal (VDH)
	vdhgeo	Vertical Dusk Horizontal (VDH)	-->	Geographical (GEO)

III- MATHEMATICAL EXPRESSIONS OF TRANSFORMATION MATRIX

1)general remarks on transformation matrix

To obtain the matrix to transform a vector expressed in a coordinate system A into another system B, the simplest way is to express the directions of the 3 axis of system B in the coordinate system A.

Indeed, if we notes the coordinates of these 3 unit axes **XB**, **YB**, **ZB** in coordinate system A as:

$$XB = \begin{pmatrix} X1 \\ X2 \\ X3 \end{pmatrix}_{(A)} \qquad YB = \begin{pmatrix} Y1 \\ Y2 \\ Y3 \end{pmatrix}_{(A)} \qquad ZB = \begin{pmatrix} Z1 \\ Z2 \\ Z3 \end{pmatrix}_{(A)}$$

any vector V can be expressed in system B as:

$$V_1^{(B)} = \mathbf{X_B} \bullet \mathbf{V} = X_1^{(A)} V_1^{(A)} + X_2^{(A)} V_2^{(A)} + X_3^{(A)} V_3^{(A)}$$
$$V_2^{(B)} = \mathbf{Y_B} \bullet \mathbf{V} = Y_1^{(A)} V_1^{(A)} + Y_2^{(A)} V_2^{(A)} + Y_3^{(A)} V_3^{(A)}$$
$$V_3^{(B)} = \mathbf{Z_B} \bullet \mathbf{V} = Z_1^{(A)} V_1^{(A)} + Z_2^{(A)} V_2^{(A)} + Z_3^{(A)} V_3^{(A)}$$

Thus the transform matrix of any vector **V** is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(B)} = \begin{pmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \\ Z1 & Z2 & Z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(A)}$$

Similarly the transformation from system B to A is:

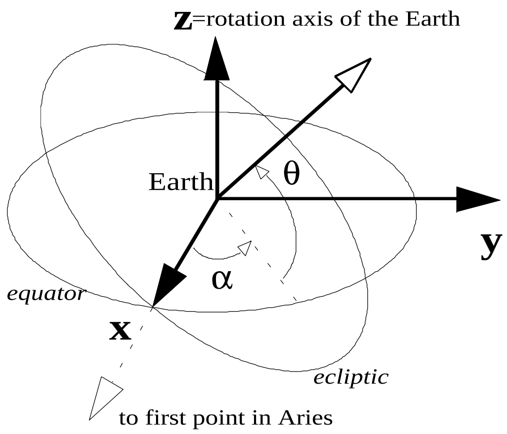
$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(A)} = \begin{pmatrix} X1 & Y1 & Z1 \\ X2 & Y2 & Z2 \\ X3 & Y3 & Z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(B)}$$

All transformation matrix have the following properties, useful for error checking:

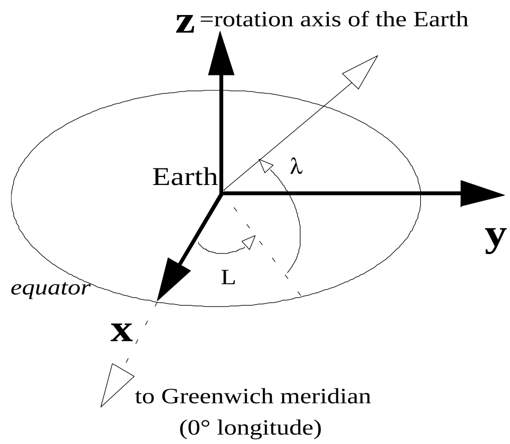
1) Each row and column is a unit vector;
2) The dot products of any two rows or any two columns is zero;
3) The cross product of any two rows or columns equals the third row or column or its negative (Row 1 cross row 2 equals row3; row 2 cross row 1 equal minus row3).

2) GEI to GEO transformations

Geocentric Equatorial Inertial system



Geographic system



GEO and GEI system have their Z-axis in common, so the only difference is a rotation around Z-axis of θ angle, thus the matrix transformation is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)}$$

and the inverse transformation is obviously:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)}$$

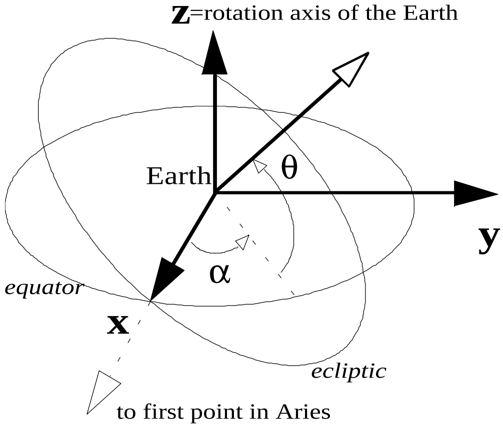
The θ angle is the angle between the Greenwich meridian and the first point in Aries, measured Eastward, in the Earth's equator, from the first point in Aries.

θ is called Greenwich Mean Sideral Time; GMST is a function of the time of the day and the time of year, since the sideral day (duration of a day relative to inertial space) is less than 24h.

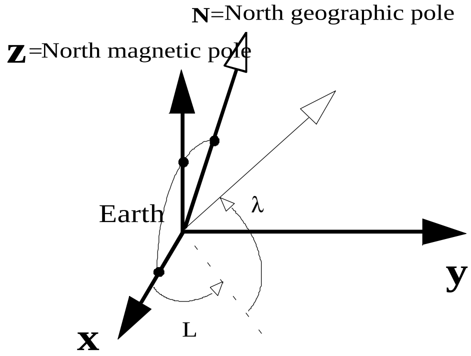
Practically, GMST is computed from CSUNDI subroutine.

3) GEI to MAG transformations

Geocentric Equatorial Inertial



Geomagnetic



Transformation from GEI to MAG system requires a knowledge of the dipole direction in GEI system, noted as **M**.

The geographic coordinates of the dipole axis can be known, for instance for IGRF epoch 1965, as 11.435° colatitude and -69.761° east longitude, thus:

$$\mathbf{D}=(D1,\ D2,\ D3)=(0.06859, -0.18602,\ 0.98015)$$

Practically, D is computed for a given time and year from CDIPDI subroutine.

To know the **M**=**D** vector in GEI system, which is the Z-axis of MAG system, we use the GEO to GEI transformation computed § III-2, so:

$$\mathbf{Z} = \mathbf{M} = \mathbf{D}_{GEI} = \begin{pmatrix} D1\ \cos\theta - D2\ \sin\theta \\ D1\ \sin\theta + D2\ \cos\theta \\ D3 \end{pmatrix}$$

where θ is the Greenwich Mean Sideral Time computed from CSUNDI subroutine.

We can deduce then the Y-axis of SM system in GEI coordinates from the cross product between North geographic pole (**N**) and North magnetic pole (**M**) :

$$\mathbf{Y} = \mathbf{N} \times \mathbf{M} \ / \ |\mathbf{N} \times \mathbf{M}|$$

Normalizing factors occurs because **N** and **M** are not necessarily perpendicular; since **N** has two components equal to zero, cross product is easy and we found:

$$\mathbf{Y} = \begin{pmatrix} -M2 \\ M1 \\ 0 \end{pmatrix} .1 \ / \left(M1^2 + M2^2\right)^{1/2}$$

X-axis is deduced from:

$$\mathbf{X} = \mathbf{Y} \times \mathbf{M}$$

so:

$$\mathbf{X} = \begin{pmatrix} M1M3 \\ M2M3 \\ -(M1^2 + M2^2) \end{pmatrix} .1 / \left(M1^2 + M2^2\right)^{1/2}$$

All coordinates of X-Y-Z axis of MAG system in GEI coordinates being known, the transform matrix of any vector **V** is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)} = \begin{pmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \\ M1 & M2 & M3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)}$$

Similarly the transformation from system MAG to GEI is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)} = \begin{pmatrix} X1 & Y1 & M1 \\ X2 & Y2 & M2 \\ X3 & Y3 & M3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)}$$

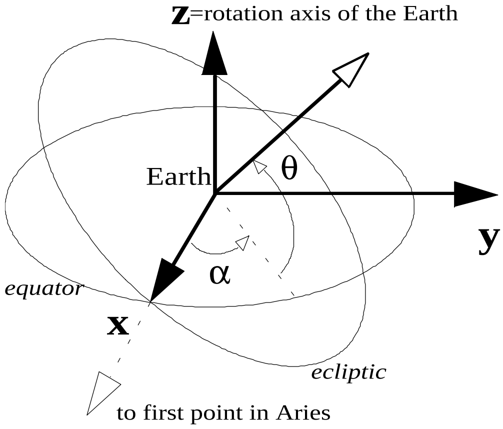
By replacing corresponding values, with $Q=(M1^2 + M2^2)^{1/2}$, we can obtain the fully expanded expression:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)} = \begin{pmatrix} (D1D3\cos\theta - D2D3\sin\theta)/Q & (D1D3\sin\theta + D2D3\cos\theta)/Q & -Q \\ -(D2\cos\theta + D1\sin\theta)/Q & (-D2\sin\theta + D1\cos\theta)/Q & 0 \\ D1\cos\theta - D2\sin\theta & D1\sin\theta + D2\cos\theta & D3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)}$$

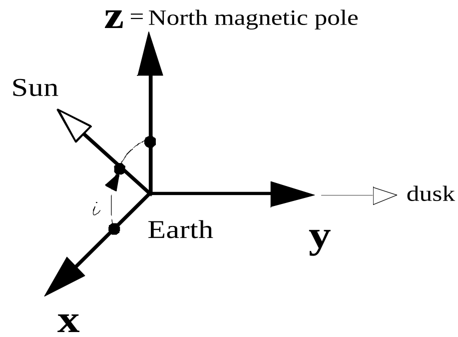
$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)} = \begin{pmatrix} (D1D3\cos\theta - D2D3\sin\theta)/Q & -(D2\cos\theta + D1\sin\theta)/Q & D1\cos\theta - D2\sin\theta \\ (D1D3\sin\theta + D2D3\cos\theta)/Q & (-D2\sin\theta + D1\cos\theta)/Q & D1\sin\theta + D2\cos\theta \\ -Q & 0 & D3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)}$$

4) GEI to SM transformations

Geocentric Equatorial Inertial



Solar Magnetic



Transformation from GEI system to SM system requires a knowledge of Sun direction and magnetic dipole direction in GEI system.

In GEI system, the direction of the Sun is computed from CSUNDI subroutine:

$$\mathbf{S}=(S1,\ S2,\ S3)$$

The geographic coordinates of the dipole axis can be known, for instance for IGRF epoch 1965, as 11.435° colatitude and -69.761° east longitude, thus:

$$\mathbf{D}=(D1,\ D2,\ D3)=(0.06859, -0.18602,\ 0.98015)$$

Practically, D is computed for a given time and year from CDIPDI subroutine.

To know the **M**=**D** vector in GEI system, which is the Z-axis of SM system, we use the GEO to GEI transformation computed § III-2, so:

$$\mathbf{Z} = \mathbf{M} = \mathbf{D}_{GEI} = \begin{pmatrix} D1\cos\theta - D2\sin\theta \\ D1\sin\theta + D2\cos\theta \\ D3 \end{pmatrix}$$

where θ is the Greenwich Mean Sideral Time computed from CSUNDI subroutine.

We can deduce then the Y-axis of SM system in GEI coordinates as:

$$\mathbf{Y} = \mathbf{M} \times \mathbf{S} \,/\, |\mathbf{M} \times \mathbf{S}|$$

(normalizing factors occurs because **M** and **S** are not necessarily perpendicular)

X-axis is deduced from:

$$\mathbf{X} = \mathbf{Y} \times \mathbf{M}$$

All coordinates of X-Y-Z axis of SM system in GEI coordinates being known, the transform matrix of any vector **V** is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)} = \begin{pmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \\ Z1 & Z2 & Z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)}$$

Similarly the transformation from system SM to GEI is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)} = \begin{pmatrix} X1 & Y1 & Z1 \\ X2 & Y2 & Z2 \\ X3 & Y3 & Z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)}$$

with respectively:

$$\begin{pmatrix} Z1 \\ Z2 \\ Z3 \end{pmatrix} = \begin{pmatrix} M1 \\ M2 \\ M3 \end{pmatrix}$$

$$\begin{pmatrix} Y1 \\ Y2 \\ Y3 \end{pmatrix} = \begin{pmatrix} M2S3 - M3S2 \\ M3S1 - M1S3 \\ M1S2 - M2S1 \end{pmatrix} .1/Q$$

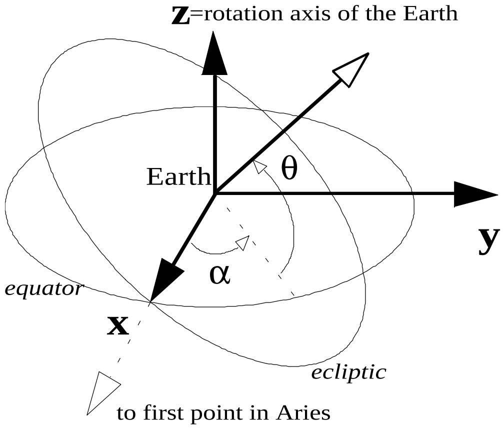
$$\begin{pmatrix} X1 \\ X2 \\ X3 \end{pmatrix} = \begin{pmatrix} Y2M3 - Y3M2 \\ Y3M1 - Y1M3 \\ Y1M2 - Y2M1 \end{pmatrix}$$

$$\begin{pmatrix} M1 \\ M2 \\ M3 \end{pmatrix} = \begin{pmatrix} D1\cos\theta - D2\sin\theta \\ D1\sin\theta + D2\cos\theta \\ D3 \end{pmatrix}$$

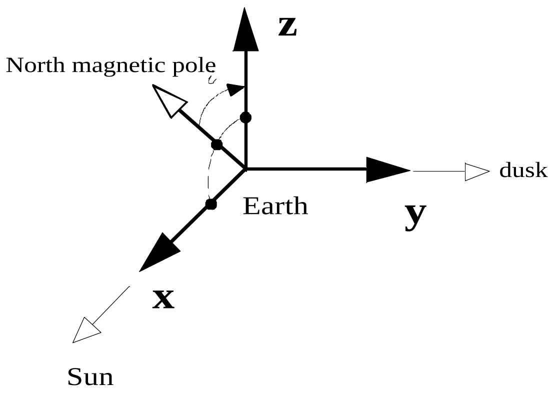
$$Q = \left[(M2S3 - M3S2)^2 + (M3S1 - M1S3)^2 + (M1S2 - M2S1)^2 \right]^{1/2}$$

5) GEI to GSM transformations

Geocentric Equatorial Inertial



Geocentric Solar Magnetospheric



Transformation from GEI system to GSM system requires a knowledge of Sun direction and magnetic dipole direction in GEI system.

In GEI system, the direction of X-axis of GSM system is the direction of the Sun computed from CSUNDI subroutine:

$$\mathbf{X}=\mathbf{S}=(S_1,\ S_2,\ S_3)$$

The geographic coordinates of the dipole axis can be known, for instance for IGRF epoch 1965, as 11.435° colatitude and -69.761° east longitude, thus:

$$\mathbf{D}=(D_1,\ D_2,\ D_3)=(0.06859, -0.18602,\ 0.98015)$$

Practically, D is computed for a given time and year from CDIPDI subroutine.

To know the **M**=**D** vector in GEI system, we use the GEO to GEI transformation computed in § III-2, so:

$$\mathbf{M} = \mathbf{D}_{GEI} = \begin{pmatrix} D_1\cos\theta - D_2\sin\theta \\ D_1\sin\theta + D_2\cos\theta \\ D_3 \end{pmatrix}$$

where θ is the Greenwich Mean Sideral Time computed from CSUNDI subroutine.

We can deduce then the Y-axis and Z-axis of GSM system in GEI coordinates as:

$$\mathbf{Y} = \mathbf{M} \times \mathbf{S} \,/\, |\mathbf{M} \times \mathbf{S}|$$

(normalizing factors occurs because **M** and **S** are not necessarily perpendicular) and

$$\mathbf{Z} = \mathbf{S} \times \mathbf{Y}$$

All coordinates of X-Y-Z axis of GSM system in GEI coordinates being known, the transform matrix of any vector **V** is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSM)} = \begin{pmatrix} S1 & S2 & S3 \\ Y1 & Y2 & Y3 \\ Z1 & Z2 & Z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{GEI}$$

Similarly the transformation from system GSM to GEI is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)} = \begin{pmatrix} S1 & Y1 & Z1 \\ S2 & Y2 & Z2 \\ S3 & Y3 & Z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{GSM}$$

with respectively:

$$\begin{pmatrix} Y1 \\ Y2 \\ Y3 \end{pmatrix} = \begin{pmatrix} M2S3 & - & M3S2 \\ M3S1 & - & M1S3 \\ M1S2 & - & M2S1 \end{pmatrix} .1/Q$$

$$\begin{pmatrix} Z1 \\ Z2 \\ Z3 \end{pmatrix} = \begin{pmatrix} S2Y3 & - & S3Y2 \\ S3Y1 & - & S1Y3 \\ S1Y2 & - & S2Y1 \end{pmatrix}$$

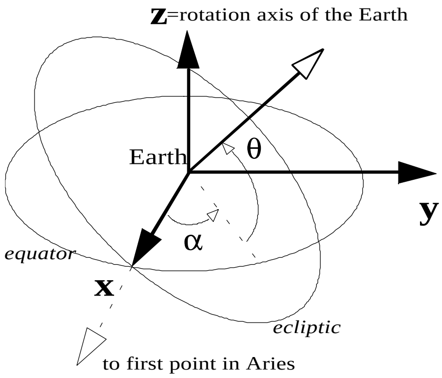
where

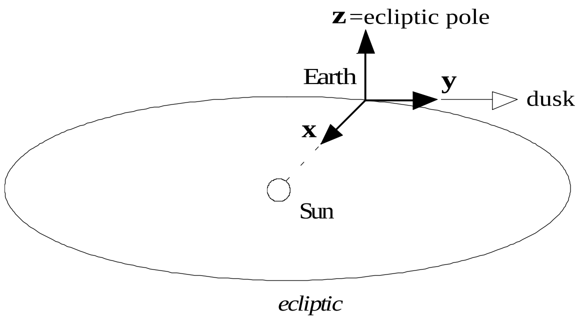
$$\begin{pmatrix} M1 \\ M2 \\ M3 \end{pmatrix} = \begin{pmatrix} D1\cos\theta & - & D2\sin\theta \\ D1\sin\theta & + & D2\cos\theta \\ & D3 & \end{pmatrix}$$

$$Q = \left[(M2S3 - M3S2)^2 + (M3S1 - M1S3)^2 + (M1S2 - M2S1)^2 \right]^{1/2}$$

6) GEI to GSE transformations

Geocentric Equatorial Inertial *Geocentric Solar Ecliptic*





In GEI system, the direction of X-axis of GSE system is the direction **S** of the SUN , computed from CSUNDI subroutine:

$$\mathbf{X}=\mathbf{S}=(S1,\ S2,\ S3)$$

The direction of the Z-axis of GSE is the direction of ecliptic pole, which is a known constant value:

$$\mathbf{Z}=\mathbf{E}=(E1,\ E2,\ E3) = (0,\ -0.398,\ 0.917)$$

The third axis, Y, is deduced from $\mathbf{Y}= \mathbf{Z} \times \mathbf{X} = \mathbf{E} \times \mathbf{S}$, thus:

$$\mathbf{Y} = \mathbf{E} \times \mathbf{S} = \begin{pmatrix} E2S3 - E3S2 \\ E3S1 - E1S3 \\ E1S2 - E2S1 \end{pmatrix}$$

Thus the transform matrix of any vector **V** is:

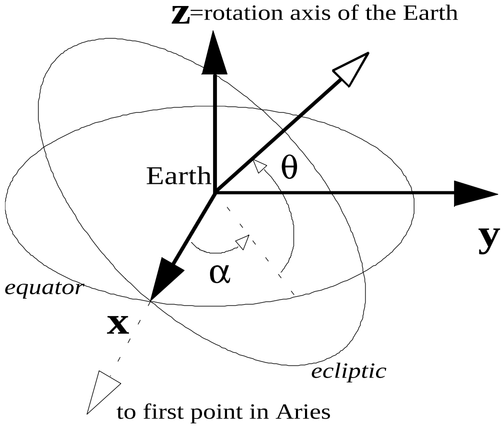
$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)} = \begin{pmatrix} S1 & S2 & S3 \\ Y1 & Y2 & Y3 \\ E1 & E2 & E3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)}$$

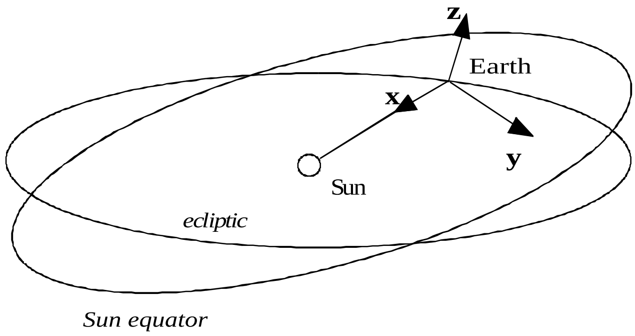
Similarly the transformation from system GSE to GEI is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)} = \begin{pmatrix} S1 & Y1 & E1 \\ S2 & Y2 & E2 \\ S3 & Y3 & E3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)}$$

7) GEI to GSEQ transformations

Geocentric Equatorial Inertial ***Geocentric Solar Equatorial***





In GEI system, the direction of X-axis of GSEQ system is the direction of the SUN computed from CSUNDI subroutine:

$$\mathbf{X}=\mathbf{S}=(S1,\ S2,\ S3)$$

The direction of the rotation axis of the SUN in GEI system is a known constant value:

$$\mathbf{R}=(R1,\ R2,\ R3) = (0.122,\ -0.424,\ 0.899)$$

Since Y-axis of GSEQ is parallel to the Sun's equatorial plane, the direction of Y-axis in GEI is $\mathbf{R} \times \mathbf{S}$; nevertheless the cross product must be normalized to have a Y unit axis, because $\mathbf{R}$ and $\mathbf{S}$ are not necessarily perpendicular, so:

$$\mathbf{Y}= (\mathbf{R} \times \mathbf{S})/|\mathbf{R} \times \mathbf{S}|$$

and $\mathbf{Z}= \mathbf{S} \times \mathbf{Y}$

Thus the transform matrix of any vector $\mathbf{V}$ is:

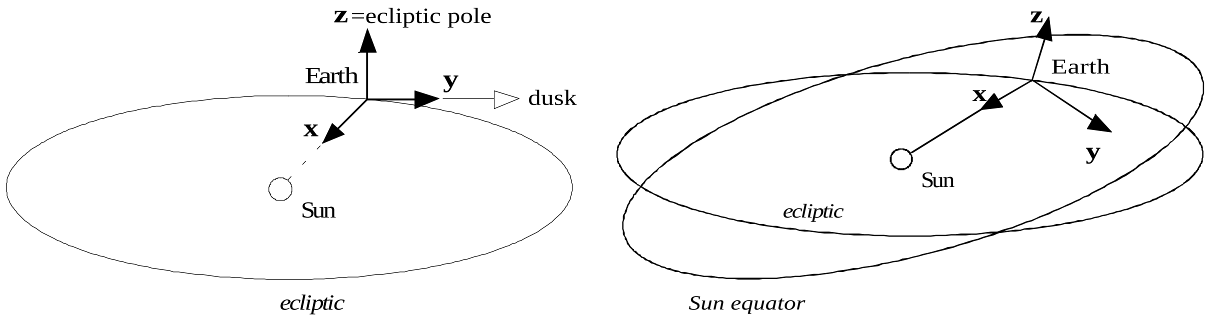
$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSEQ)} = \begin{pmatrix} S1 & S2 & S3 \\ Y1 & Y2 & Y3 \\ Z1 & Z2 & Z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)}$$

Similarly the transformation from system GSEQ to GEI is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)} = \begin{pmatrix} S1 & Y1 & Z1 \\ S2 & Y2 & Z2 \\ S3 & Y3 & Z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSEQ)}$$

8) GSE to GSEQ transformations

Geocentric Solar Ecliptic ***Geocentric Solar Equatorial***



The only difference between GSE and GSEQ systems is a rotation about the common X-axis, to have the Y-GSEQ axis parallel to the Sun equator plane.

So, if θ is the rotation angle, the transformation matrix of any vector V is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSEQ)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)}$$

and the inverse transformation is obviously:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSEQ)}$$

<u>computation of θ angle:</u>

θ is the $(\mathbf{Y}_{GSE},\mathbf{Y}_{GSEQ})$ angle, so $\sin\theta = |\mathbf{Y}_{GSE} \times \mathbf{Y}_{GSEQ}|$

To compute θ, we use the following known vectors in GEI system:

1) the direction **S** of the SUN , computed from CSUNDI subroutine:

$$\mathbf{S}=(S1,\ S2,\ S3)$$

2) the direction of ecliptic pole, which is a known constant value:

$$\mathbf{E}=(E1,\ E2,\ E3) = (0,\ -0.398,\ 0.917)$$

3) the direction of the rotation axis of the Sun, which is also a constant value:

$$\mathbf{R}= (R1,\ R2,\ R3) = (0.122,\ -0.424,\ 0.899)$$

To compute $\sin\theta = |\mathbf{Y}_{GSE} \times \mathbf{Y}_{GSEQ}|$ we use the following properties:

$$\mathbf{Y}_{GSE} = \mathbf{Z}_{GSE} \times \mathbf{X}_{GSE} = \mathbf{E} \times \mathbf{S}$$

and since R is in the X-Z plane in the GSEQ system:

$$\mathbf{Y}_{GSEQ} = (\mathbf{R} \times \mathbf{S})/|\mathbf{R} \times \mathbf{S}|$$

so we have $\sin\theta = |(\mathbf{E} \times \mathbf{S}) \times (\mathbf{R} \times \mathbf{S})| / |\mathbf{R} \times \mathbf{S}|$

since $(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \times \mathbf{B}\bullet\mathbf{D})\mathbf{C} - (\mathbf{A} \times \mathbf{B}\bullet\mathbf{C})\mathbf{D}$

and $(\mathbf{A} \times \mathbf{B})\bullet\mathbf{C} = (\mathbf{A}\bullet\mathbf{B}) \times \mathbf{C}$

thus $(\mathbf{E} \times \mathbf{S}) \times (\mathbf{R} \times \mathbf{S}) = (\mathbf{E} \times \mathbf{S}\bullet\mathbf{S})\mathbf{R} - (\mathbf{E} \times \mathbf{S}\bullet\mathbf{R})\mathbf{S} = (\mathbf{R} \times \mathbf{E}\bullet\mathbf{S})\mathbf{S}$

and finally, as **S** is a unit vector:

$$\boxed{\sin\theta = (\mathbf{R} \times \mathbf{E})\bullet\mathbf{S} \ / \ |\mathbf{R} \times \mathbf{S}|}$$

For numerical applications, expression

$$\sin\theta = (\mathbf{R} \times \mathbf{E}) \bullet \mathbf{S} \;/\; |\mathbf{R} \times \mathbf{S}|$$

can be extended as:

$$\sin\theta = \frac{[(R_2E_3-R_3E_2)S_1 + (R_3E_1-R_1E_3)S_2 + (R_1E_2-R_2E_1)S_3]}{[(R_2S_3-R_3S_2)^2 + (R_3S_1-R_1S_3)^2 + (R_1S_2-R_2S_1)^2]^{1/2}}$$

with numerical values above, this becomes:

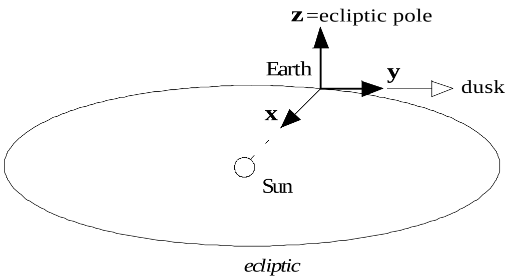
$$\sin\theta = (-0.031,\ -0.112,\ -0.049) \bullet \mathbf{S} \;/\; |(0.122,\ -0.424,\ 0.899) \times \mathbf{S}|$$

Since the Sun's spin axis is inclined 7.25° to the ecliptic, θ changes from -7.25 to 7.25 each year, from approximately December 5 to June 5.

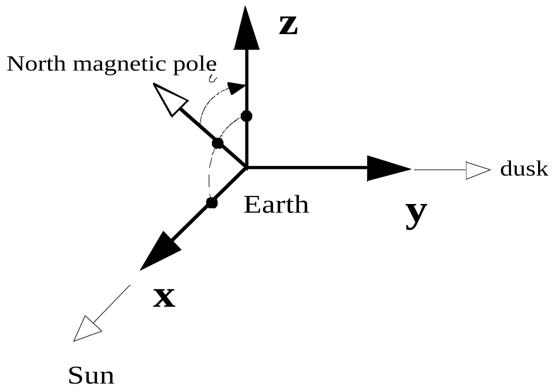
The Sun's spin axis is directed most towards the Earth on approximately September 5 at which time the Earth reaches its northern most heliographic latitude. At this time θ=0.

9) GSE to GSM transformations

Geocentric Solar Ecliptic



Geocentric Solar Magnetospheric



GSE and GSM systems have their X-axis in common, so the only difference is a rotation around the X-axis of the angle ξ, thus the matrix transformation is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSM)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\xi & \sin\xi \\ 0 & -\sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)}$$

and the inverse transformation is obviously:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\xi & -\sin\xi \\ 0 & \sin\xi & \cos\xi \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSM)}$$

Nevertheless the ξ angle cannot be obtained from a simple equation.

To compute the rotation terms of transformation matrix, we use the GSE to GEI and the GEI to GSM previous matrix transformations, given in § III-6 and III-5.

These transformations are noted (see § III-6 and III-5):

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)} = \begin{pmatrix} S1 & S2 & S3 \\ y1 & y2 & y3 \\ E1 & E2 & E3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)}$$

with $\mathbf{y} = (\mathbf{E} \times \mathbf{S})$ in GEI system

and:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEI)} = \begin{pmatrix} S1 & Y1 & Z1 \\ S2 & Y2 & Z2 \\ S3 & Y3 & Z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSM)}$$

with $\mathbf{Y} = (\mathbf{M} \times \mathbf{S})$ in GEI system
and $\mathbf{Z} = (\mathbf{S} \times \mathbf{Y})$ in GEI system

we can write the GSM to GSE transformation as:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)} = \begin{pmatrix} S1 & S2 & S3 \\ y1 & y2 & y3 \\ E1 & E2 & E3 \end{pmatrix} \begin{pmatrix} S1 & Y1 & Z1 \\ S2 & Y2 & Z2 \\ S3 & Y3 & Z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSM)}$$

which give

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)} = \begin{pmatrix} \mathbf{S} \bullet \mathbf{S} & \mathbf{S} \bullet \mathbf{Y} & \mathbf{S} \bullet \mathbf{Z} \\ \mathbf{y} \bullet \mathbf{S} & \mathbf{y} \bullet \mathbf{Y} & \mathbf{y} \bullet \mathbf{Z} \\ \mathbf{E} \bullet \mathbf{S} & \mathbf{E} \bullet \mathbf{Y} & \mathbf{E} \bullet \mathbf{Z} \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSM)}$$

which becomes, since **S** and **Y** are unit vectors perpendicular between us, as **S** and **Z**:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathbf{y} \bullet \mathbf{Y} & \mathbf{y} \bullet \mathbf{Z} \\ 0 & \mathbf{E} \bullet \mathbf{Y} & \mathbf{E} \bullet \mathbf{Z} \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSM)}$$

Of course the final matrix does not depend on the **S** vector.

Computation of $\cos\xi$

We have to equalize $\mathbf{y}\bullet\mathbf{Y}$ and $\mathbf{E}\bullet\mathbf{Z}$ terms as $\cos\xi$.

Taking $\mathbf{y}\bullet\mathbf{Y}=(\mathbf{E}\times\mathbf{S})\bullet(\mathbf{M}\times\mathbf{S})$

we compute $\mathbf{E}\bullet\mathbf{Z}=\mathbf{E}\bullet(\mathbf{S}\times\mathbf{Y})=\mathbf{E}\bullet[(\mathbf{S}\times(\mathbf{M}\times\mathbf{S})]$

since $\mathbf{A}\bullet(\mathbf{B}\times\mathbf{C})=(\mathbf{A}\times\mathbf{B})\bullet\mathbf{C}$

we effectively found $\mathbf{E}\bullet\mathbf{Z}=(\mathbf{E}\times\mathbf{S})\bullet(\mathbf{M}\times\mathbf{S})=\mathbf{y}\bullet\mathbf{Y}$

then:

$$\mathbf{y}\bullet\mathbf{Y}=\mathbf{E}\bullet\mathbf{Z}=\cos\xi$$

by replacing the corresponding values, we set:

$$\cos\xi=(\mathbf{E}\times\mathbf{S})\bullet(\mathbf{M}\times\mathbf{S})$$

since $\mathbf{A}\bullet(\mathbf{B}\times\mathbf{C})=(\mathbf{A}\times\mathbf{B})\bullet\mathbf{C}$

we found $\cos\xi=-(\mathbf{E}\times\mathbf{S})\bullet(\mathbf{S}\times\mathbf{M})=-(\mathbf{E}\times\mathbf{S})\times\mathbf{S}$

and finally

$$\cos\xi=\mathbf{E}\bullet\mathbf{M}$$

E and **M** are known since:

1) the direction of ecliptic pole in GEI system is a known constant value:

$$\mathbf{E}=(E1,\ E2,\ E3)=(0,\ -0.398,\ 0.917)$$

2) **M** is the dipole direction in GEI system, computed § III-5 as:

$$\mathbf{M}=(M1,\ M2,\ M3)=\left((D_1\cos\theta-D_2\sin\theta),\ (D_1\sin\theta+D_2\cos\theta),\ D_3\right)$$

3) the geographic coordinates of the dipole axis **D** is computed for a given time and year from CDIPDI subroutine; for instance for IGRF epoch 1965 we have:

$$\mathbf{D}=(D1,\ D2,\ D3)=(0.06859, -0.18602,\ 0.98015)$$

4) the θ Greenwich Mean Sideral Time is computed for a given time and year from CSUNDI subroutine.

This leads to the final expanded formula:

$$\cos\xi=E_1(D_1\cos\theta-D_2\sin\theta)+E_2(D_1\sin\theta+D_2\cos\theta)+E_3D_3$$

Computation of $\sin\xi$

Similarly one has to ensure that the $\mathbf{E}\bullet\mathbf{Y}$ and $-\mathbf{y}\bullet\mathbf{Z}$ terms are equal.

Taking $\mathbf{y}\bullet\mathbf{Z}=(\mathbf{E}\times\mathbf{S})\bullet(\mathbf{S}\times\mathbf{Y})$

since $\mathbf{A}\bullet(\mathbf{B}\times\mathbf{C})=(\mathbf{A}\times\mathbf{B})\bullet\mathbf{C}$

we have $\mathbf{y}\bullet\mathbf{Z}=[(\mathbf{E}\times\mathbf{S})\times\mathbf{S}]\bullet\mathbf{Y}$

and find:

$$\mathbf{y}\bullet\mathbf{Z}=-\mathbf{E}\bullet\mathbf{Y}=-\sin\xi$$

Similarly computation of $\sin\xi$ is made as:

$$\sin\xi=\mathbf{E}\bullet\mathbf{Y}$$

thus $\sin\xi=\mathbf{E}\bullet(\mathbf{M}\times\mathbf{S})$

yet $\sin\xi=E_1(M_2S_3-M_3S_2)+E_2(M_3S_1-M_1S_3)+E_3(M_1S_2-M_2S_1)$

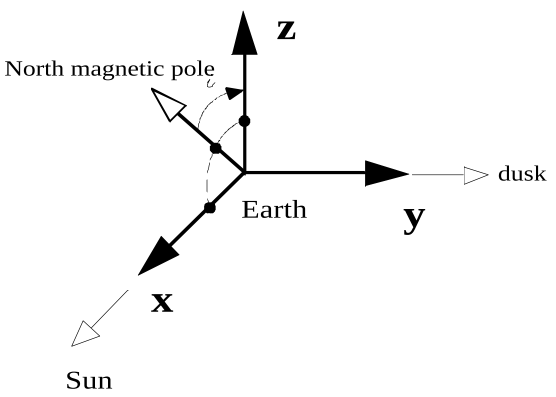
since $\mathbf{M}=\left((D_1\cos\theta-D_2\sin\theta)\,,\,(D_1\sin\theta+D_2\cos\theta)\,,\,D_3\right)$

finally we have:

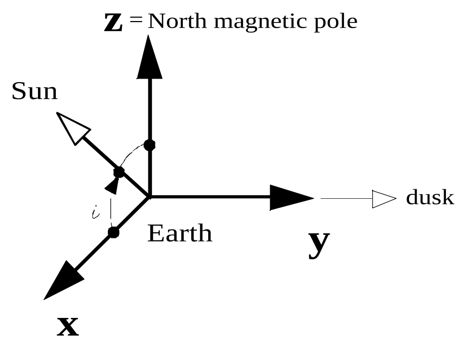
$$\begin{aligned}\sin\xi=\;&E_1\left[(D_1\sin\theta+D_2\cos\theta)S3-D_3S2\right]\\+\;&E_2\left[D_3S1-(D_1\cos\theta-D_2\sin\theta)S3\right]\\+\;&E_3\left[(D_1\cos\theta-D_2\sin\theta)S2-(D_1\sin\theta+D_2\cos\theta)S1\right]\end{aligned}$$

10) GSM to SM transformations

Geocentric Solar Magnetospheric



Solar Magnetic



The GSM and SM system have the Y-axis in common, then the transformation matrix is a simple rotation of μ angle, which is named the dipole tilt angle.

Thus the transformation matrix of any vector V is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)} = \begin{pmatrix} \cos\mu & 0 & -\sin\mu \\ 0 & 1 & 0 \\ \sin\mu & 0 & \cos\mu \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSM)}$$

and the inverse transformation is obviously:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSM)} = \begin{pmatrix} \cos\mu & 0 & \sin\mu \\ 0 & 1 & 0 \\ -\sin\mu & 0 & \cos\mu \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)}$$

μ can be obtained from $\sin\mu = \mathbf{S} \bullet \mathbf{D}$, where $\mathbf{S}$ is the direction of the sun and $\mathbf{D}$ the dipole direction, both in GEO system for instance.

s can be computed from GEI to GEO transformation given in § III-2; then we have:

$$s=(s_1,\ s_2,\ s_3)=\left((S_1\cos\theta+S_2\sin\theta),\ (-S_1\sin\theta+S_2\cos\theta),\ S_3\right)$$

where S is the direction of the Sun in GEI system, computed from CSUNDI subroutine, such as the Greenwich Mean Sideral Time θ.

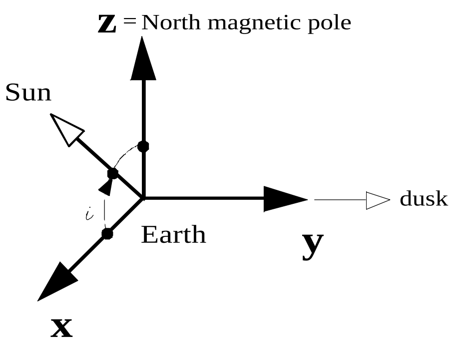
D is obtained from the International Geomagnetic Reference Field (IGRF); practically, **D** is computed for a given time and year from CDIPDI subroutine; value for 1965.0 is:

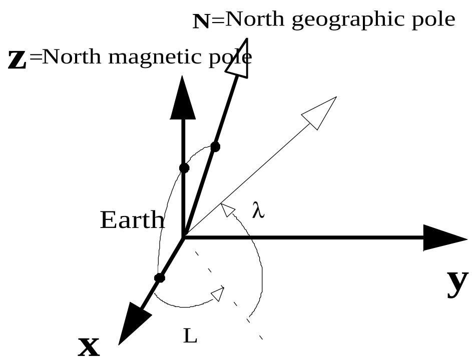
$$\mathbf{D}=(D_1,\ D_2,\ D_3)=(0.06859, -0.18602,\ 0.98015)$$

finally, rotation matrix elements are:

$$\sin\mu = s_1D_1 + s_2D_2 + s_3D_3$$
$$\cos\mu = (1-\sin^2\mu)^{1/2}$$

11) SM to MAG transformations





The SM and MAG system have the Z-axis in common, then the transformation matrix is a simple rotation of φ angle, thus the transformation matrix of any vector V is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)}$$

and the inverse transformation is obviously:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)} = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)}$$

Nevertheless the angle φ is not derivable from a simple equation.

To compute the rotation terms of transformation matrix, we use the MAG to GEO and the GEO to SM matrix transformations, given in § III-12 and III-13.

The first transformation is noted (see § III-12):

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)} = \begin{pmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \\ D1 & D2 & D3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)}$$

with $\mathbf{X}=(\mathbf{Y} \times \mathbf{D})$ in GEO System
and $\mathbf{Y}=(\mathbf{N} \times \mathbf{D})/|\mathbf{N} \times \mathbf{D}|$

N and **D** vector are respectively the North geographic pole and the magnetic dipole in geographic system.

Practically, geographic dipole direction **D** is computed for a given time and year from CDIPDI subroutine; value for 1965.0 is:

$$\mathbf{D}=(D1,\ D2,\ D3)=(0.06859, -0.18602,\ 0.98015)$$

we deduce from $\mathbf{Y}=(\mathbf{N} \times \mathbf{D})/|\mathbf{N} \times \mathbf{D}|$:

$$\mathbf{Y}=(-D2,\ D1,\ 0\ )/(D_1^2+D_2^2)^{1/2}$$

The second transformation is noted (see § III-13):

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)} = \begin{pmatrix} x1 & y1 & D1 \\ x2 & y2 & D2 \\ x3 & y3 & D3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)}$$

with $\mathbf{x}=(\mathbf{y} \times \mathbf{D})$ in GEO system
$\mathbf{y}=(\mathbf{D} \times \mathcal{S})/|\mathbf{D} \times \mathcal{S}|$

$\mathcal{S}$ is the direction of the sun in GEO system, which can be computed from GEI to GEO transformation given in § III-2, then we have:

$$\mathcal{S}=(\mathcal{S}1,\ \mathcal{S}2,\ \mathcal{S}3) = \left((S_1\cos\theta + S_2\sin\theta)\ ,\ (-S_1\sin\theta + S_2\cos\theta)\ ,\ S_3\right)$$

S is the direction of the Sun in GEI system, computed from CSUNDI subroutine, such as the Greenwich Mean Sideral Time θ.

Knowing all elements for MAG to GEO and GEO to SM transformations, we can write the SM to MAG transformation as:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)} = \begin{pmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \\ D1 & D2 & D3 \end{pmatrix} \begin{pmatrix} x1 & y1 & D1 \\ x2 & y2 & D2 \\ x3 & y3 & D3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)}$$

which give

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)} = \begin{pmatrix} \mathbf{X} \bullet \mathbf{x} & \mathbf{X} \bullet \mathbf{y} & \mathbf{X} \bullet \mathbf{D} \\ \mathbf{Y} \bullet \mathbf{x} & \mathbf{Y} \bullet \mathbf{y} & \mathbf{Y} \bullet \mathbf{D} \\ \mathbf{D} \bullet \mathbf{x} & \mathbf{D} \bullet \mathbf{y} & \mathbf{D} \bullet \mathbf{D} \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)}$$

In other hand we can write the following equivalences:

$$\mathbf{X}\bullet\mathbf{D} = (\mathbf{Y} \times \mathbf{D})\bullet\mathbf{D} = \mathbf{Y}\bullet(\mathbf{D} \times \mathbf{D}) = 0$$
$$\mathbf{Y}\bullet\mathbf{D} = (\mathbf{N} \times \mathbf{D})\bullet\mathbf{D}/|\mathbf{N} \times \mathbf{D}| = \mathbf{N}\bullet(\mathbf{D} \times \mathbf{D})/|\mathbf{N} \times \mathbf{D}| = 0$$

$$\mathbf{D}\bullet\mathbf{x} = \mathbf{D}\bullet(\mathbf{y} \times \mathbf{D}) = -\mathbf{D}\bullet(\mathbf{D} \times \mathbf{y}) = -(\mathbf{D} \times \mathbf{D})\bullet\mathbf{y} = 0$$
$$\mathbf{D}\bullet\mathbf{y} = \mathbf{D}\bullet(\mathbf{D} \times \mathcal{S})/|\mathbf{D} \times \mathcal{S}| = (\mathbf{D} \times \mathbf{D})\bullet\mathcal{S}/|\mathbf{D} \times \mathcal{S}| = 0$$
$$\mathbf{D}\bullet\mathbf{D} = 1$$

then we have the following matrix:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)} = \begin{pmatrix} \mathbf{X} \bullet \mathbf{x} & \mathbf{X} \bullet \mathbf{y} & 0 \\ \mathbf{Y} \bullet \mathbf{x} & \mathbf{Y} \bullet \mathbf{y} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)}$$

Computation of cos φ

We have to equalize $\mathbf{X}\bullet\mathbf{x}$ and $\mathbf{Y}\bullet\mathbf{y}$ terms as $\cos\varphi$.

Taking $$\mathbf{X}\bullet\mathbf{x} = (\mathbf{Y} \times \mathbf{D})\bullet(\mathbf{y} \times \mathbf{D}) = -(\mathbf{Y} \times \mathbf{D})\bullet(\mathbf{D} \times \mathbf{y})$$

$$\mathbf{X}\bullet\mathbf{x} = -(\mathbf{Y} \times \mathbf{D}) \times \mathbf{D}\bullet\mathbf{y}$$

and since $\mathbf{Y}$ and $\mathbf{D}$ are perpendicular:

$$\boxed{\mathbf{X}\bullet\mathbf{x} = \mathbf{Y}\bullet\mathbf{y} = \cos\varphi}$$

coordinates of $\mathbf{Y}$ axe is given in § III-12 as:

$$\mathbf{Y} = (-D2,\ D1,\ 0\ )/(D_1^2+D_2^2)^{1/2}$$

and $$\mathbf{y} = (\mathbf{D} \times \delta)/|\mathbf{D} \times \delta|$$

then we have:

$$\cos\varphi = \mathbf{Y}\bullet\mathbf{y} = \mathbf{Y}\bullet(\mathbf{D} \times \delta)/|\mathbf{D} \times \delta| = (\mathbf{Y} \times \mathbf{D})\bullet\delta/|\mathbf{D} \times \delta|$$

The $(\mathbf{Y} \times \mathbf{D})$ vector can be expanded as:

$$(\mathbf{Y} \times \mathbf{D}) = \left(D1D3,\ D2D3,\ -(D_1^2+D_2^2)\right)/(D_1^2+D_2^2)^{1/2}$$

and we deduce:

$$\boxed{\cos\varphi = \left[\left(D1D3\delta 1 + D2D3\delta 2 - (D_1^2+D_2^2)\delta 3\right)\right]/Q}$$

with

$$Q = (D_1^2+D_2^2)^{1/2} \cdot \left[(D2\delta 3 - D3\delta 2)^2 + (D3\delta 1 - D1\delta 3)^2 + D1\delta 2 - D2\delta 1)^2\right]^{1/2}$$

$$(\delta 1,\ \delta 2,\ \delta 3) = \left((S_1\cos\theta + S_2\sin\theta)\ ,\ (-S_1\sin\theta + S_2\cos\theta)\ ,\ S_3\right)$$

Computation of $\sin\varphi$

We have to equalize $\mathbf{X}\bullet\mathbf{y}$ and $-\mathbf{Y}\bullet\mathbf{x}$ terms as $\sin\varphi$.

Taking $\mathbf{X}\bullet\mathbf{y}=(\mathbf{Y} \times \mathbf{D})\bullet\mathbf{y}$

and $-\mathbf{Y}\bullet\mathbf{x}= -\mathbf{Y}\bullet(\mathbf{y} \times \mathbf{D}) = \mathbf{Y}\bullet(\mathbf{D} \times \mathbf{y})= (\mathbf{Y} \times \mathbf{D})\bullet\mathbf{y}$

yet we have well:

$$\boxed{\mathbf{X}\bullet\mathbf{y} = -\mathbf{Y}\bullet\mathbf{x} = \sin\varphi}$$

$\sin\varphi$ is then computed from $\sin\varphi= (\mathbf{Y} \times \mathbf{D})\bullet\mathbf{y}$

from above we have:

$$(\mathbf{Y} \times \mathbf{D})=\left(D_1D_3,\ D_2D_3,\ -(D_1^2+D_2^2)\right)/(D_1^2+D_2^2)^{1/2}$$

and

$$\mathbf{y}=(\mathbf{D} \times \mathscr{S})/|\mathbf{D} \times \mathscr{S}|$$

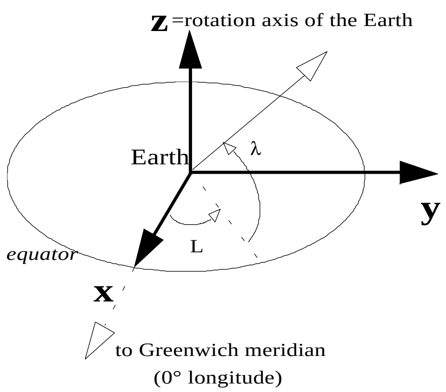
we deduce:

$$\boxed{\sin\varphi = \left(D_2\mathscr{S}_1 - D_1\mathscr{S}_2\right)/Q}$$

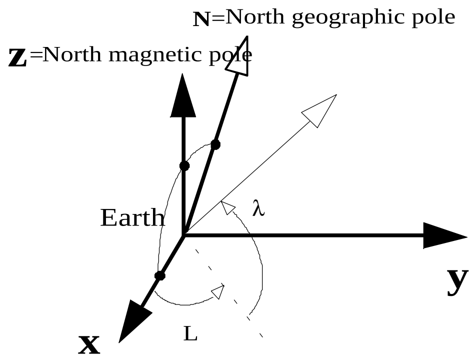
with Q such as above.

12) GEO to MAG transformations

Geographic



Geomagnetic



In geomagnetic coordinates, Z-axis is parallel to the magnetic dipole axis D, and the Y-axis is perpendicular to the North geographic pole N, then we have in geographic system:

$$\mathbf{Z} = \mathbf{D}$$

$$\mathbf{Y} = (\mathbf{N} \times \mathbf{D})/|\mathbf{N} \times \mathbf{D}|$$

and $$\mathbf{X} = \mathbf{Y} \times \mathbf{D}$$

The geographic coordinates of the dipole axis can be known, for instance for IGRF epoch 1965, as 11.435° colatitude and -69.761° east longitude.

Practically, geographic dipole direction **D** is computed for a given time and year from CDIPDI subroutine; value for 1965.0 is:

$$\mathbf{D}=(D1,\ D2,\ D3)=(0.06859, -0.18602,\ 0.98015)$$

we deduce:

$$\mathbf{Y}=(-D_2,\ D_1,\ 0\ )/(D_1{}^2+D_2{}^2)^{1/2}$$

and:

$$\mathbf{X}=(Y_2D_3,\ -Y_1D_3,\ Y_1D_2-Y_2D_1)$$

Thus the transform matrix of any vector **V** is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)} = \begin{pmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \\ D1 & D2 & D3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)}$$

Similarly the transformation from system MAG to GEO is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)} = \begin{pmatrix} X1 & Y1 & D1 \\ X2 & Y2 & D2 \\ X3 & Y3 & D3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)}$$

For IGRF epoch 1965.0, we have for instance:

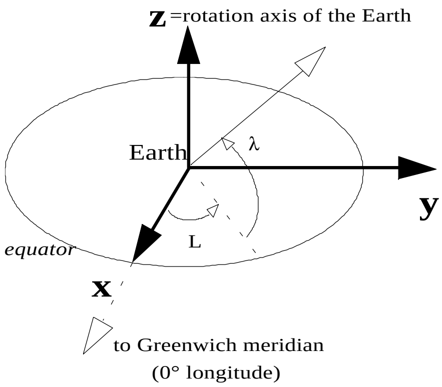
$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)} = \begin{pmatrix} 0.33908 & -0.91963 & -0.19826 \\ 0.93825 & 0.34595 & 0. \\ 0.06859 & -0.18602 & 0.98015 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)}$$

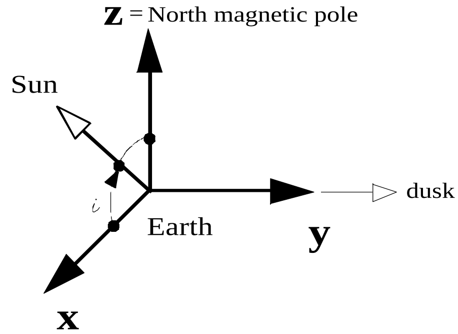
and similarly:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)} = \begin{pmatrix} 0.33908 & 0.93825 & 0.06859 \\ -0.91963 & 0.34595 & -0.18602 \\ -0.19826 & 0. & 0.98015 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(MAG)}$$

13) GEO to SM transformations

Geographic ***Solar Magnetic***





In GEO system, the direction of the Z-axis of SM system is the dipole direction **D**. The geographic coordinates of the dipole axis can be known, for instance for IGRF epoch 1965, as 11.435° colatitude and -69.761° east longitude, thus:

$$\mathbf{D}=(D1,\ D2,\ D3)=(0.06859, -0.18602,\ 0.98015)$$

Practically, D is computed for a given time and year from CDIPDI subroutine.

We can then deduce in GEO system the Y-axis of SM system as:

$$\mathbf{y}=(\ \mathbf{D}\ \times\ \mathcal{S})/|\ \mathbf{D}\ \times\ \mathcal{S}|$$

where $\mathcal{S}$ is the direction of the sun in GEO system, which can be computed from GEI to GEO transformation given in § III-2, then we have:

$$\mathcal{S}=(\mathcal{S}1,\ \mathcal{S}2,\ \mathcal{S}3)\ =\big((S_1\cos\theta + S_2\sin\theta)\ ,\ (-S_1\sin\theta + S_2\cos\theta)\ ,\ S_3\big)$$

S is the direction of the Sun in GEI system, computed from CSUNDI subroutine, such as the Greenwich Mean Sideral Time θ; normalizing factors occurs because **D** and $\mathcal{S}$ are not necessarily perpendicular.

The third axis X is computed from:

$$\mathbf{x}=(\ \mathbf{y}\ \times\ \mathbf{D})$$

and the GEO to SM transformation is given by:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)} = \begin{pmatrix} x1 & x2 & x3 \\ y1 & y2 & y3 \\ D1 & D2 & D3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)}$$

Similarly the SM to GEO transformation is given by:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)} = \begin{pmatrix} x1 & y1 & D1 \\ x2 & y2 & D2 \\ x3 & y3 & D3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(SM)}$$

with respectively:

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} y2D3 & - & y3D2 \\ y3D1 & - & y1D3 \\ y1D2 & - & y2D1 \end{pmatrix}$$

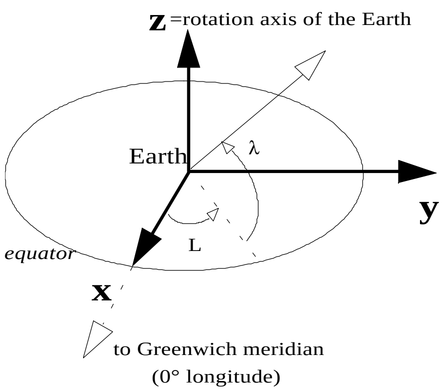
$$\begin{pmatrix} y1 \\ y2 \\ y3 \end{pmatrix} = \begin{pmatrix} D2\mathcal{S}3 & - & D3\mathcal{S}2 \\ D3\mathcal{S}1 & - & D1\mathcal{S}3 \\ D1\mathcal{S}2 & - & D2\mathcal{S}1 \end{pmatrix} .1/Q$$

$$Q = \left[(D2\mathcal{S}3 - D3\mathcal{S}2)^2 + (D3\mathcal{S}1 - D1\mathcal{S}3)^2 + (D1\mathcal{S}2 - D2\mathcal{S}1)^2 \right]^{1/2}$$

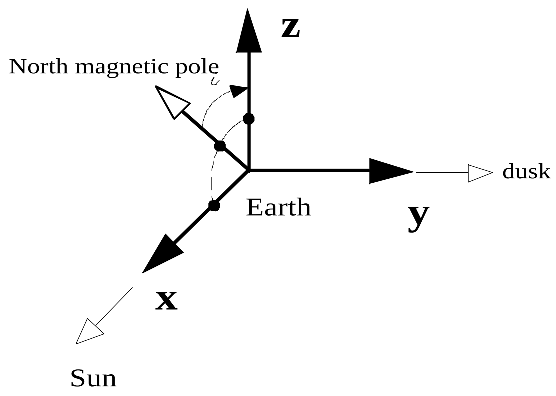
$$\mathcal{S} = (\mathcal{S}1, \mathcal{S}2, \mathcal{S}3) = \left((S_1\cos\theta + S_2\sin\theta), (-S_1\sin\theta + S_2\cos\theta), S_3 \right)$$

14) GEO to GSM transformations

Geographic



Geocentric Solar Magnetospheric



In GEO system, the direction of the X-axis of GSM system is the direction of the sun $\mathbf{S}$, which can be computed from GEI to GEO transformation given in § III-2; then we have:

$$\mathbf{x}=\mathbf{S}=(S_1,\ S_2,\ S_3)=\left((S_1\cos\theta+S_2\sin\theta),\ (-S_1\sin\theta+S_2\cos\theta),\ S_3\right)$$

S is the direction of the Sun in GEI system, computed from CSUNDI subroutine, such as the Greenwich Mean Sideral Time θ.

The Y axis in GEO system can be deduced from:

$$\mathbf{y}=(\mathbf{D}\times\mathbf{S})/|\mathbf{D}\times\mathbf{S}|$$

where **D** is the dipole direction; the geographic coordinates of the dipole axis can be known, for instance for IGRF epoch 1965, as 11.435° colatitude and -69.761° east longitude, thus:

$$\mathbf{D}=(D_1,\ D_2,\ D_3)=(0.06859,-0.18602,\ 0.98015)$$

Practically, D is computed for a given time and year from CDIPDI subroutine; normalizing factors in cross product occurs because **D** and $\mathbf{S}$ are not necessarily perpendicular.

And finally the third axis Z is computed from:

$$\mathbf{z}=\mathbf{S}\times\mathbf{y}$$

and the GEO to GSM transformation is given by:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSM)} = \begin{pmatrix} x1 & x2 & x3 \\ y1 & y2 & y3 \\ z1 & z2 & z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)}$$

Similarly the GSM to GEO transformation is given by:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)} = \begin{pmatrix} x1 & y1 & z1 \\ x2 & y2 & z2 \\ x3 & y3 & z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSM)}$$

with respectively:

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} \mathscr{S}1 \\ \mathscr{S}2 \\ \mathscr{S}3 \end{pmatrix}$$

$$\begin{pmatrix} y1 \\ y2 \\ y3 \end{pmatrix} = \begin{pmatrix} D2\mathscr{S}3 & - & D3\mathscr{S}2 \\ D3\mathscr{S}1 & - & D1\mathscr{S}3 \\ D1\mathscr{S}2 & - & D2\mathscr{S}1 \end{pmatrix} .1/Q$$

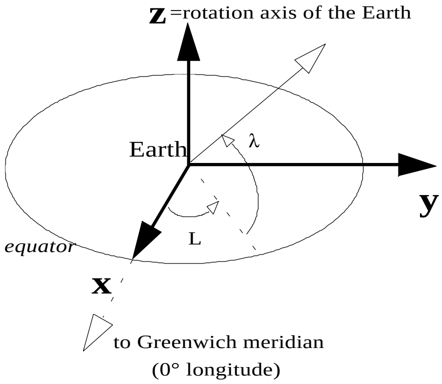
$$\begin{pmatrix} z1 \\ z2 \\ z3 \end{pmatrix} = \begin{pmatrix} x2y3 & - & x3y2 \\ x3y1 & - & x1y3 \\ x1y2 & - & x2y1 \end{pmatrix}$$

$$Q = \left[(D2\mathscr{S}3 - D3\mathscr{S}2)^2 + (D3\mathscr{S}1 - D1\mathscr{S}3)^2 + (D1\mathscr{S}2 - D2\mathscr{S}1)^2 \right]^{1/2}$$

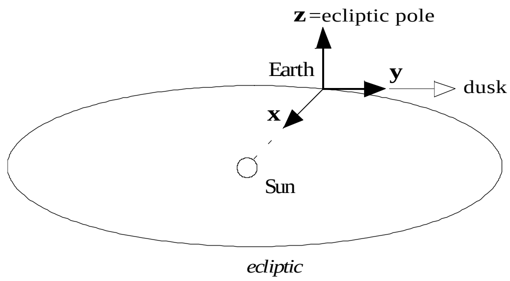
$$\mathscr{S} = (\mathscr{S}1, \mathscr{S}2, \mathscr{S}3) = \Big((S_1\cos\theta + S_2\sin\theta), (-S_1\sin\theta + S_2\cos\theta), S_3 \Big)$$

15) GEO to GSE transformations

Geographic



Geocentric Solar Ecliptic



In GEO system, the direction of the X-axis of GSE system is the direction of the sun $\mathcal{S}$, which can be computed from GEI to GEO transformation given in § III-2; then we have:

$$\mathbf{X}=\mathcal{S}=(\mathcal{S}1,\ \mathcal{S}2,\ \mathcal{S}3)=\left((S_1\cos\theta+S_2\sin\theta),\ (-S_1\sin\theta+S_2\cos\theta),\ S_3\right)$$

S is the direction of the Sun in GEI system, computed from CSUNDI subroutine, such as the Greenwich Mean Sideral Time θ.

The Z axis is the direction of the ecliptic pole, which is a known constant value in GEI system:

$$\mathbf{E}=(E1,\ E2,\ E3)=(0,\ -0.398,\ 0.917)$$

from GEI to GEO transformation we have:

$$\mathbf{Z}=\mathcal{E}=(\mathcal{E}1,\ \mathcal{E}2,\ \mathcal{E}3)=\left((E_1\cos\theta+E_2\sin\theta),\ (-E_1\sin\theta+E_2\cos\theta),\ E_3\right)$$

And finally the Y axis in GEO system can be deduced from:

$$\mathbf{Y}=\mathcal{E}\times\mathcal{S}$$

and the GEO to GSE transformation is given by:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)} = \begin{pmatrix} \mathcal{S}1 & \mathcal{S}2 & \mathcal{S}3 \\ Y1 & Y2 & Y3 \\ \mathcal{E}1 & \mathcal{E}2 & \mathcal{E}3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)}$$

Similarly the GSE to GEO transformation is given by:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)} = \begin{pmatrix} \mathcal{S}1 & Y1 & \mathcal{E}1 \\ \mathcal{S}2 & Y2 & \mathcal{E}2 \\ \mathcal{S}3 & Y3 & \mathcal{E}3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSE)}$$

with respectively:

$$\begin{pmatrix} Y1 \\ Y2 \\ Y3 \end{pmatrix} = \begin{pmatrix} \mathcal{E}2\mathcal{S}3 - \mathcal{E}3\mathcal{S}2 \\ \mathcal{E}3\mathcal{S}1 - \mathcal{E}1\mathcal{S}3 \\ \mathcal{E}1\mathcal{S}2 - \mathcal{E}2\mathcal{S}1 \end{pmatrix}$$

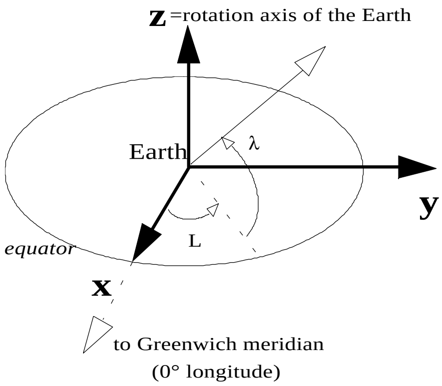
$$\mathcal{S}=(\mathcal{S}1,\ \mathcal{S}2,\ \mathcal{S}3) = \left((S_1\cos\theta + S_2\sin\theta)\ ,\ (-S_1\sin\theta + S_2\cos\theta)\ ,\ S_3\right)$$

$$\mathcal{E}=(\mathcal{E}1,\ \mathcal{E}2,\ \mathcal{E}3) = \left((E_1\cos\theta + E_2\sin\theta)\ ,\ (-E_1\sin\theta + E_2\cos\theta)\ ,\ E_3\right)$$

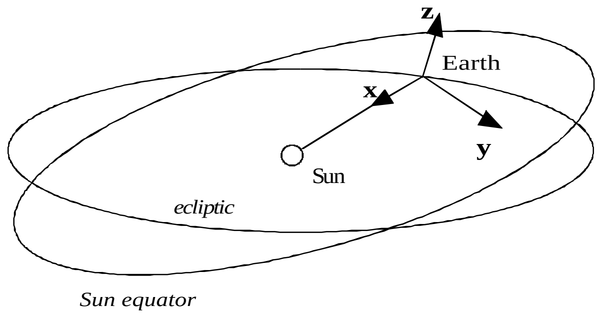
$$\mathbf{E}=(E1,\ E2,\ E3) = (0,\ -0.398,\ 0.917)$$

16) GEO to GSEQ transformations

Geographic



Geocentric Solar Equatorial



In GEO system, the direction of the X-axis of GSEQ system is the direction of the sun $\mathcal{S}$, which can be computed from GEI to GEO transformation given in § III-2; then we have:

$$\mathbf{X}=\mathcal{S}=(\mathcal{S}1,\ \mathcal{S}2,\ \mathcal{S}3) = \big((S_1\cos\theta + S_2\sin\theta),\ (-S_1\sin\theta + S_2\cos\theta),\ S_3\big)$$

S is the direction of the Sun in GEI system, computed from CSUNDI subroutine, such as the Greenwich Mean Sideral Time θ.

The Sun equator plane is defined from the direction of the rotation axis of the SUN which is in GEI system a known constant value:

$$\mathbf{R}=(R1,\ R2,\ R3) = (0.122,\ -0.424,\ 0.899)$$

from GEI to GEO transformation we obtain this vector in GEO system as:

$$\mathcal{R}=(\mathcal{R}1,\ \mathcal{R}2,\ \mathcal{R}3) = \big((R_1\cos\theta + R_2\sin\theta),\ (-R_1\sin\theta + R_2\cos\theta),\ R_3\big)$$

Then the Y axis in GEO system can be deduced from:

$$\mathbf{y}= \mathcal{R} \times \mathcal{S} / | \mathcal{R} \times \mathcal{S} |$$

normalizing factors in cross product occurs because $\mathcal{R}$ and $\mathcal{S}$ are not necessarily perpendicular.

The third axis Z is computed from:

$$\mathbf{z}= \mathcal{S} \times \mathbf{y}$$

and the GEO to GSEQ transformation is given by:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSEQ)} = \begin{pmatrix} x1 & x2 & x3 \\ y1 & y2 & y3 \\ z1 & z2 & z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)}$$

Similarly the GEQ to GEO transformation is given by:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)} = \begin{pmatrix} x1 & y1 & z1 \\ x2 & y2 & z2 \\ x3 & y3 & z3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GSEQ)}$$

with respectively:

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} = \begin{pmatrix} \mathcal{S}1 \\ \mathcal{S}2 \\ \mathcal{S}3 \end{pmatrix}$$

$$\begin{pmatrix} y1 \\ y2 \\ y3 \end{pmatrix} = \begin{pmatrix} \mathcal{R}2\mathcal{S}3 - \mathcal{R}3\mathcal{S}2 \\ \mathcal{R}3\mathcal{S}1 - \mathcal{R}1\mathcal{S}3 \\ \mathcal{R}1\mathcal{S}2 - \mathcal{R}2\mathcal{S}1 \end{pmatrix} .1/Q$$

$$\begin{pmatrix} z1 \\ z2 \\ z3 \end{pmatrix} = \begin{pmatrix} \mathcal{S}2y3 - \mathcal{S}3y2 \\ \mathcal{S}3y1 - \mathcal{S}1y3 \\ \mathcal{S}1y2 - \mathcal{S}2y1 \end{pmatrix}$$

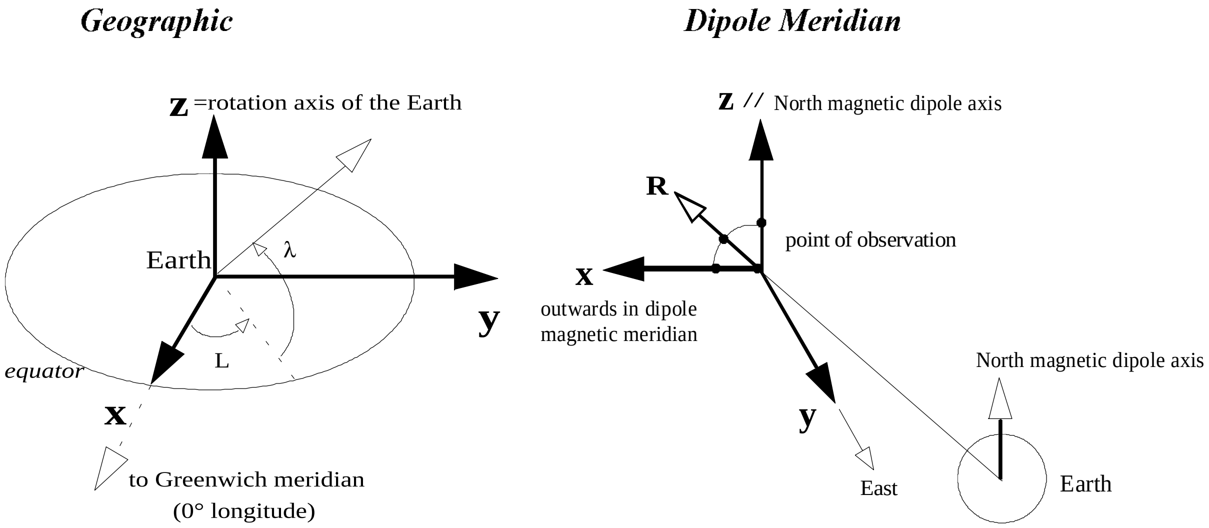
$$Q = \left[(\mathcal{R}2\mathcal{S}3 - \mathcal{R}3\mathcal{S}2)^2 + (\mathcal{R}3\mathcal{S}1 - \mathcal{R}1\mathcal{S}3)^2 + (\mathcal{R}1\mathcal{S}2 - \mathcal{R}2\mathcal{S}1)^2 \right]^{1/2}$$

$$\mathcal{S} = (\mathcal{S}1, \mathcal{S}2, \mathcal{S}3) = \left((S_1\cos\theta + S_2\sin\theta), (-S_1\sin\theta + S_2\cos\theta), S_3 \right)$$

$$\mathcal{R} = (\mathcal{R}1, \mathcal{R}2, \mathcal{R}3) = \left((R_1\cos\theta + R_2\sin\theta), (-R_1\sin\theta + R_2\cos\theta), R_3 \right)$$

$$\mathbf{R} = (R1, R2, R3) = (0.122, -0.424, 0.899)$$

17) GEO to DM transformations



Dipole meridian system is a local coordinate system, and varies with the position of the point of observation relative to the centre of the Earth; this positions is noted in GEO system as:

$$\mathbf{R}=(R1,\ R2,\ R3)$$

To transform GEO coordinate to DM coordinates, we need the dipole position in GEO system which is the Z axis of DM system. The geographic coordinates of the dipole axis can be known, for instance for IGRF epoch 1965, as 11.435° colatitude and -69.761° east longitude, thus:

$$\mathbf{Z} = \mathbf{D} = (D1,\ D2,\ D3)=(0.06859,-0.18602,\ 0.98015)$$

Practically, D is computed for a given time and year from CDIPDI subroutine.

We can deduce then the Y-axis of DM system in GEO coordinates as:

$$\mathbf{Y}= \mathbf{D} \times \mathbf{R} \ / \ |\mathbf{D} \times \mathbf{R}|$$

normalizing factors occurs because **D** and **R** are not necessarily perpendicular and because **R** is not a unit vector.

The third axis X is deduced from:

$$\mathbf{X}= \mathbf{Y} \times \mathbf{D}$$

All coordinates of X-Y-Z axis of DM system in GEO coordinates being known, the transform matrix of any vector **V** is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(DM)} = \begin{pmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \\ D1 & D2 & D3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)}$$

Similarly the transformation from system DM to GEO is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)} = \begin{pmatrix} X1 & Y1 & D1 \\ X2 & Y2 & D2 \\ X3 & Y3 & D3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(DM)}$$

with respectively:

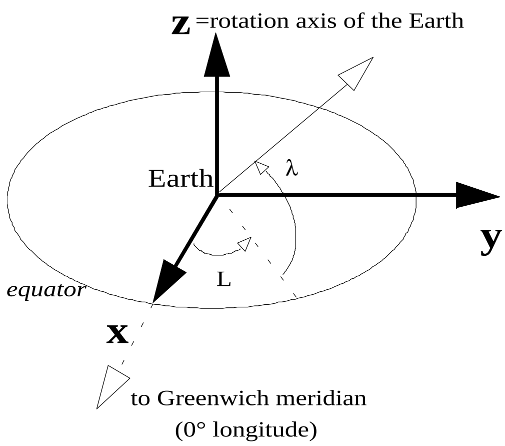
$$\begin{pmatrix} X1 \\ X2 \\ X3 \end{pmatrix} = \begin{pmatrix} Y2D3 - Y3D2 \\ Y3D1 - Y1D3 \\ Y1D2 - Y2D1 \end{pmatrix}$$

$$\begin{pmatrix} Y1 \\ Y2 \\ Y3 \end{pmatrix} = \begin{pmatrix} D2R3 - D3R2 \\ D3R1 - D1R3 \\ D1R2 - D2R1 \end{pmatrix} .1/Q$$

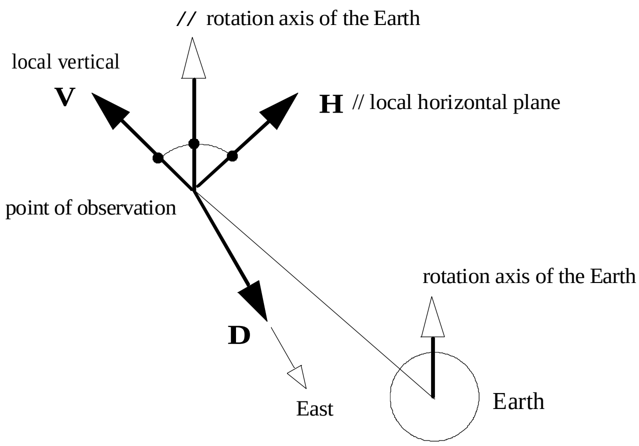
$$Q = [(D2R3 - D3R2)^2 + (D3R1 - D1R3)^2 + (D1R2 - D2R1)^2]^{1/2}$$

18) GEO to VDH transformations

Geographic



VDH



VDH system is a local coordinate system, and varies with the position of the point of observation relative to the centre of the Earth; this positions is noted in GEO system as:

$$\mathbf{R}=(R1,\ R2,\ R3)$$

and we have directly

$$\mathbf{V}=\mathbf{R}/(R1^2 + R2^2 + R3^2)^{1/2}$$

Since D is perpendicular to V and to the rotation axis of the Earth, we have:

$$\mathbf{D}= \mathbf{Z_E} \times \mathbf{R} \ / \ |\mathbf{Z_E} \times \mathbf{R}|$$

which give:

$$\mathbf{D} = \begin{pmatrix} -R2 \\ R1 \\ 0 \end{pmatrix} . 1/(R1^2 + R2^2)^{1/2}$$

The third axis H is deduced from:

$$\mathbf{H}= \mathbf{V} \times \mathbf{D}$$

which give:

$$\mathbf{H} = \begin{pmatrix} -R1R3 \\ -R2R3 \\ R1^2 + R2^2 \end{pmatrix} . 1/Q$$

with

$$Q=[(R1^2 + R2^2)(R1^2 + R2^2 + R3^2)]^{1/2}$$

All coordinates of X-Y-Z axis of VDH system in GEO coordinates being known, the transform matrix of any vector **V** is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(VDH)} = \begin{pmatrix} V1 & V2 & V3 \\ D1 & D2 & D3 \\ H1 & H2 & H3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)}$$

Similarly the transformation from system VDH to GEO is:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(GEO)} = \begin{pmatrix} V1 & D1 & H1 \\ V2 & D2 & H2 \\ V3 & D3 & H3 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \end{pmatrix}_{(VDH)}$$

IV- CALENDAR CONVERSIONS

1) general remarks

Most space missions deliver or use for their processing data date and time in various format, such as decimal Julian day, month-day in the month, year, decimal hours, hour-min-sec, and so on. In order to easily manipulate all theses quantities, and to get transformations between else, special calendar conversions, described below, has been added to ROCOT library.

2) list of calendar conversions

name	*work*
cleapy	compute if a given year is or not a leap year.
codoty	compute day of the year from given month, day in the month and year.
cdfdot	compute date as month, day in the month, from given day of the year and year
cjud50	compute Julian day from 1/1/1950 from given month, day in the month and year.
cdfjud	compute date as month, day in the month and year from given 1950 Julian day.
codech	compute decimal hour from given hour, minute and second.
cotime	compute hour, minute and second from given decimal hour.

3) definition of conversions

compute if a given year is or not a leap year (cleapy)

A given year (as 1990) is a leap year if this number is divisible by 4; if the year is a secular year (as 1900 or 2000), this year is a leap year if this number is divisible by 400.

For instance, 1988, 1992, 2000 are leap years, 1900, 1989, 2100 are not.

compute day of the year from given month, day in the month and year (codoty)

this conversion compute the day of the year, equal to 1 for January 1, for a date given as month, day in the month, and year.

For instance, the day of the year of October 17, 1990 is 290. This conversion counts the number of the day in a month, taking into account leap year since February has not a constant number of days (using cleapy).

compute date as month, day in the month, from given day of the year and year (cdfdot)

This is the inverse conversion of codoty, since one compute the corresponding month and day in the month for a given day of the year, and a given year. Year is necessary to know if the year is or not a leap year (using cleapy).

compute Julian day from given month, day in the month and year (cjud50)

From a given date as month, day, year, this conversion compute the Julian day , i.e. day number from January 1, 1950, with Julian day equal to 1 for January 1, 1950 (use codoty and cleapy conversions).

compute date as month, day in the month and year from given 1950 Julian day (cdfjud)

This is the inverse conversion of cjud50, since one compute for a given Julian day the corresponding date as month, day in the month and year (use cleapy and cdfdot conversions).

compute decimal hour from given hour, minute and second (codech)

This conversion makes the operation hour + minute/60. + second/3600.

compute hour, minute and second from given decimal hour (cotime)

Inverse of codech.

V- ROCOT LIBRARY

1) general remarks

ROCOT library delivers 4 kinds of subroutines:

a) "Read and check" subroutines, such as `rxxxxx` (redate, retime), which query and read on the terminal date and time, with automatic check of good format (no hour greater than 23, or minutes greater than 59, and so on), useful to write an interactive user program where date and time are an input.

b) "Computation" subroutines, such as `cxxxxx` (codoty, cotime etc..). that only do computations without any particular previous call. These are mainly concerned with calendar conversions and are useful for dates and times given in various format.

One has include in this category the particular '`ctimpa`' subroutine which set for a given date and time all the time varying quantities needed for the transformations matrix (results are stored in 15 common transparent to the user).

c) "Give" subroutines, such as `gxxxxx` (gsundi, gdipdi, etc...) which give in the output arguments useful parameters as direction of the Sun, direction of the dipole etc... in GEI and GEO systems; subroutine ctimpa must be called before using theses subroutines at each times where the date and time are changed;

d) "Transformations" subroutines, such as `txxxxxx` (tgeigeo, tgeogei, tmaggsm, tgsmmag, etc...) which transforms input cartesian coordinates system into an another one following mathematical expressions given in section III; as above, subroutine ctimpa must be called before using theses subroutines at each times where the date and time are changed. Note that the name have 7 characters rather 6, but it allows to avoid confusion with any equivalent subroutine in Tsyganenko magnetic field model which could be used simultaneously with Rocotlib; most of Fortran compiler accept it.

Section VI give an example of Fortran user program, directions for use, example of installation of the library on a UNIX system, and provide a test program code source to check the validity of the library.

2) description of "Read and check" subroutines

redate

```
subroutine redate(imonth,iday,iyear)
```

read_date *from terminal input and check validity*

test if imonth is not greater than 12, test if iday is not greater then length of the month, taking into account the leap years; year must be greater or equal to 1900; use cleapy subroutine.

```
input : imonth,iday,iyear
output: print error if date is not valid, and ask again
```

retime

```
subroutine retime(ih,im,is)
```

read_time *from terminal input and check validity*

ih must be between 1 and 23, im and is between 1 and 59

```
input : ih,im,is
output: print error if time is not valid, and ask again
```

3) description of "Computation" subroutines

ctimpa

```
subroutine ctimpa(iyear,idoty,ihour,imin,isec)
```

compute_time_parameters: *prepare matrix for coordinate transformations*

prepare all time varying quantities for computations of coordinate transforms of the library, and store results in 15 common statements; use csundi and cdipdi subroutines. Quantities stored in commons are below:

sin and cos of GMST
ecliptic pole in GEI system
direction of the rotation axis of the sun in GEI system
dipole direction in GEI system
direction of the sun in GEO system
direction of the ecliptic in GEO system
direction of the rotation axis of the sun in GEO system
cross product MxS in GEI system
cross product ExS in GEI system
cross product RxS in GEI system
cross product RxE in GEI system
cross product DxS in GEO system
cross product ExS in GEO system
cross product RxS in GEO system
computation of gei to mag vectors
computation of gei to sm vectors
computation of gei to gsm vectors
computation of gei to gseq vectors
computation of tetq angle
computation of mu angle
computation of dzeta angle
computation of phi angle
computation of geo to mag vectors
computation of geo to sm vectors
computation of geo to gsm vectors
computation of geo to gsq vectors

```
input :  iyear : year (1901-2099)
         idoty : day of the year (1 for January 1)
         ihour,imin,isec : hours, minutes, seconds U.T.

output:  in common statements
```

cdipdi

```
subroutine cdipdi(iyear,idoty,d1,d2,d3)
```

compute_dipole_direction *in GEO system*

compute geodipole axis direction from International Geomagnetic Reference Field (IGRF) models for time interval 1965 to 1990. For time out of interval, computation is made for nearest boundary.

```
input :  iyear : year (1965 - 1990)
output:  d1,d2,d3  cartesian dipole components in GEO
```

csundi

```
subroutine csundi(iyear,idoty,ihour,imin,isec,gst,slong,sra,sdec)
```

compute_sun_direction *in GEI system*

(from C.T. Russel,cosmic electrodynamics,v.2, 184-196, 1971)
calculates four quantities in gei system necessary for coordinate transformations dependent on sun position (and, hence, on universal time and season)

```
input :  iyear : year (1901-2099)
         idoty : day of the year (1 for January 1)
         ihour,imin,isec : hours, minutes, seconds U.T.

output: gst      Greenwich mean sideral time (radians)
        slong    longitude along ecliptic (radians)
        sra      right ascension (radians)
        sdec     declination of the sun (radians)
```

cleapy

```
subroutine cleapy(iyear,ileap)
```

compute_leap_year *with ileap=1 for leap year, 0 if not*

```
input : iyear (ex: 1980)
output: ileap (1 or 0 if iyear is or not a leap year)
```

codoty

```
subroutine codoty(imonth,iday,iyear,idoty)
```

compute_day_of_the_year *with idoty=1 for January 1*
(taking into account leap year); use cleapy.

```
input : imonth,iday,iyear    ex: 10,17,1990
output: idoty                ex: 290
```

cjud50

```
subroutine cjud50(imonth,iday,iyear,jud50)
```

compute_Julian_day *with jud50=1 for January 1, 1950 (taking account leap years); use codoty and cleapy*

```
input : imonth,iday,iyear   ex: 10,17,1990
output: jud50
```

cdfdot

```
subroutine cdfdot(iyear,idoty,imonth,iday)
```

compute_date_from_day_of_the_year *and for a given year; use cleapy.*

```
input : iyear,idoty (idoty=1 for January 1)
output: imonth,iday
```

cdfjud

```
subroutine cdfjud(jud50,imonth,iday,iyear)
```

compute_date_from_Julian_day *with jud50=1 for January 1, 1950 use cleapy and cdfdot*

```
input : jud50  Julian day (1= 1/1/1950)
output: imonth,iday,iyear
```

codech

```
subroutine codech(ih,im,is,dech)
```

compute_decimal_hour *from hours, minutes, seconds*

```
input : ih,im,is
output: dech   decimal hour
```

cotime

```
subroutine cotime(dech,ih,im,is)
```

compute_time *from decimal hour*

```
input : dech decimal hour
output: ih,im,is
```

canara

```
subroutine canara(ux,uy,uz,vx,vy,vz,angle,ratio)
```

compute_angle_and_ratio *between U and V vectors*

```
input : ux,uy,uz
        vx,vy,vz

output: angle=angle between U and V (radians)
        ratio= mod(U)/mod(V)
```

4) description of "Give" subroutines

gsundi

```
subroutine gsundi(sxgei,sygei,szgei,sxgeo,sygeo,szgeo)
```

give_sun_direction *in GEI and GEO system*

```
input :  none
output:  sxgei,sygei,szgei  cartesian sun GEI coord.
         sxgeo,sygeo,szgeo  cartesian sun GEO coord.
```

gdipdi

```
subroutine gdipdi(dxgei,dygei,dzgei,dxgeo,dygeo,dzgeo)
```

give_dipole_direction *in GEI and GEO system*

```
input :  none
output:  dxgei,dygei,dzgei  cartesian dipole GEI coord.
         dxgeo,dygeo,dzgeo  cartesian dipole GEO coord.
```

gecldi

```
subroutine gecldi(exgei,eygei,ezgei,exgeo,eygeo,ezgeo)
```

give_ecliptic_direction *in GEI and GEO system*

```
input :  none
output:  exgei,eygei,ezgei  cartesian eclip. GEI coord.
         exgeo,eygeo,ezgeo  cartesian eclip. GEO coord.
```

gsrodi

```
subroutine gsrodi(rxgei,rygei,rzgei,rxgeo,rygeo,rzgeo)
```

give_sun_rotation_direction *in GEI and GEO system*

```
input : none
output: rxgei,rygei,rzgei cartesian sun rot. GEI coord.
        rxgeo,rygeo,rzgeo cartesian sun rot. GEO coord.
```

gsunpa

```
subroutine gsunpa(gmst,slon,sras,sdec)
```

give_sun_parameter *dependant of time in GEI system*

```
input :   none
output:   gmst     Greenwich mean sideral time(radians)
          slon     longitude along ecliptic (radians)
          sras     right ascension (radians)
          sdec     declination of the sun (radians)
```

gdipta

```
subroutine gdipta(dipta)
```

give_dipole_tilt_angle *in GSM system (radians)*

```
input :   none
output:   dipta=(D,Z) angle in GSM (radians)
           (dipta>0 when the north magnetic pole
            is tilted toward the Sun)
```

gvernu

```
subroutine gvernu(vernu)
```

give_version_number *of the library*

```
input :   none
output:   vernu (ex 1.0)
```

5) description of "Transformations" subroutines

tgeigeo

```
subroutine tgeigeo(xgei,ygei,zgei,xgeo,ygeo,zgeo)
```

transforms_gei_to_geo*: GEI -> GEO system*

```
input : xgei,ygei,zgei   cartesian gei coordinates
output: xgeo,ygeo,zgeo   cartesian geo coordinates
```

tgeogei

```
subroutine tgeogei(xgeo,ygeo,zgeo,xgei,ygei,zgei)
```

transforms_geo_to_gei*: GEO -> GEI system*

```
input : xgeo,ygeo,zgeo cartesian geo coordinates
output: xgei,ygei,zgei cartesian gei coordinates
```

tgeimag

```
subroutine tgeimag(xgei,ygei,zgei,xmag,ymag,zmag)
```

transforms_gei_to_mag*: GEI -> MAG system*

```
input : xgei,ygei,zgei   cartesian gei coordinates
output: xmag,ymag,zmag   cartesian mag coordinates
```

tmaggei

```
subroutine tmaggei(xmag,ymag,zmag,xgei,ygei,zgei)
```

transforms_mag_to_gei*: MAG -> GEI system*

```
input : xmag,ymag,zmag cartesian mag coordinates
output: xgei,ygei,zgei cartesian gei coordinates
```

tgeisma

```
subroutine tgeisma(xgei,ygei,zgei,xsma,ysma,zsma)
```

transforms_gei_to_sma*: GEI -> SM system*

```
input : xgei,ygei,zgei   cartesian gei coordinates
output: xsma,ysma,zsma   cartesian sma coordinates
```

tsmagei

```
subroutine tsmagei(xsma,ysma,zsma,xgei,ygei,zgei)
```

transforms_sma_to_gei*: SM -> GEI system*

```
input : xsma,ysma,zsma   cartesian sma coordinates
output: xgei,ygei,zgei   cartesian gei coordinates
```

tgeism

```
subroutine tgeigsm(xgei,ygei,zgei,xgsm,ygsm,zgsm)
```

transforms_gei_to_gsm*: GEI -> GSM system*

```
input : xgei,ygei,zgei   cartesian gei coordinates
output: xgsm,ygsm,zgsm   cartesian gsm coordinates
```

tgsmgei

```
subroutine tgsmgei(xgsm,ygsm,zgsm,xgei,ygei,zgei)
```

transforms_gsm_to_gei*: GSM -> GEI system*

```
input : xgsm,ygsm,zgsm   cartesian gsm coordinates
output: xgei,ygei,zgei   cartesian gei coordinates
```

tgeigse

```
subroutine tgeigse(xgei,ygei,zgei,xgse,ygse,zgse)
```

transforms_gei_to_gse*: GEI -> GSE system*

```
input : xgei,ygei,zgei cartesian gei coordinates
output: xgse,ygse,zgse cartesian gse coordinates
```

tgsegei

```
subroutine tgsegei(xgse,ygse,zgse,xgei,ygei,zgei)
```

transforms_gse_to_gei*: GSE -> GEI system*

```
input : xgse,ygse,zgse    cartesian gse coordinates
output: xgei,ygei,zgei    cartesian gei coordinates
```

tgeigsq

```
subroutine tgeigsq(xgei,ygei,zgei,xgsq,ygsq,zgsq)
```

transforms_gei_to_gsq*: GEI -> GSEQ system*

```
input : xgei,ygei,zgei   cartesian gei coordinates
output: xgsq,ygsq,zgsq   cartesian gsq coordinates
```

tgsqgei

```
subroutine tgsqgei(xgsq,ygsq,zgsq,xgei,ygei,zgei)
```

transforms_gsq_to_gei: GSEQ-> GEI system

```
input : xgsq,ygsq,zgsq   cartesian gsq coordinates
output: xgei,ygei,zgei   cartesian gei coordinates
```

tgsegsq

```
subroutine tgsegsq(xgse,ygse,zgse,xgsq,ygsq,zgsq)
```

transforms_gse_to_gsq: *GSE -> GSEQ system*

```
input : xgse,ygse,zgse   cartesian gse coordinates
output: xgsq,ygsq,zgsq   cartesian gsq coordinates
```

tgsqgse

```
subroutine tgsqgse(xgsq,ygsq,zgsq,xgse,ygse,zgse)
```

transforms_gsq_to_gse: *GSEQ-> GSE system*

```
input : xgsq,ygsq,zgsq   cartesian gsq coordinates
output: xgse,ygse,zgse   cartesian gse coordinates
```

tgsegsm

```
subroutine tgsegsm(xgse,ygse,zgse,xgsm,ygsm,zgsm)
```

transforms_gse_to_gsm: *GSE -> GSM system*

```
input : xgse,ygse,zgse   cartesian gse coordinates
output: xgsm,ygsm,zgsm   cartesian gsm coordinates
```

tgsmgse

```
subroutine tgsmgse(xgsm,ygsm,zgsm,xgse,ygse,zgse)
```

transforms_gsm_to_gse: *GSM -> GSE system*

```
input : xgsm,ygsm,zgsm   cartesian gsm coordinates
output: xgse,ygse,zgse   cartesian gse coordinates
```

tgsmsma

```
subroutine tgsmsma(xgsm,ygsm,zgsm,xsma,ysma,zsma)
```

transforms_gsm_to_sma: *GSM -> SM system*

```
input : xgsm,ygsm,zgsm   cartesian gsm coordinates
output: xsma,ysma,zsma   cartesian sma coordinates
```

tsmagsm

```
subroutine tsmagsm(xsma,ysma,zsma,xgsm,ygsm,zgsm)
```

transforms_sma_to_gsm: *SM -> GSM system*

```
input : xsma,ysma,zsma   cartesian sma coordinates
output: xgsm,ygsm,zgsm   cartesian gsm coordinates
```

tsmamag

```
subroutine tsmamag(xsma,ysma,zsma,xmag,ymag,zmag)
```

transforms_sma_to_mag*: SM -> MAG system*

```
input : xsma,ysma,zsma   cartesian sma coordinates
output: xmag,ymag,zmag   cartesian mag coordinates
```

tmagsma

```
subroutine tmagsma(xmag,ymag,zmag,xsma,ysma,zsma)
```

transforms_mag_to_sma*: MAG -> SM system*

```
input : xmag,ymag,zmag   cartesian mag coordinates
output: xsma,ysma,zsma   cartesian sma coordinates
```

tgeomag

```
subroutine tgeomag(xgeo,ygeo,zgeo,xmag,ymag,zmag)
```

transforms_geo_to_mag*: GEO -> MAG system*

```
input : xgeo,ygeo,zgeo   cartesian geo coordinates
output: xmag,ymag,zmag   cartesian mag coordinates
```

tmaggeo

```
subroutine tmaggeo(xmag,ymag,zmag,xgeo,ygeo,zgeo)
```

transforms_mag_to_geo: MAG -> GEO system

```
input : xmag,ymag,zmag   cartesian mag coordinates
output: xgeo,ygeo,zgeo   cartesian geo coordinates
```

tgeosma

```
subroutine tgeosma(xgeo,ygeo,zgeo,xsma,ysma,zsma)
```

transforms_geo_to_sma*: GEO -> SM system*

```
input : xgeo,ygeo,zgeo   cartesian geo coordinates
output: xsma,ysma,zsma   cartesian sma coordinates
```

tsmageo

```
subroutine tsmageo(xsma,ysma,zsma,xgeo,ygeo,zgeo)
```

transforms_sma_to_geo*: SM -> GEO system*

```
input : xsma,ysma,zsma   cartesian sma coordinates
output: xgeo,ygeo,zgeo   cartesian geo coordinates
```

tgeogsm

```
subroutine tgeogsm(xgeo,ygeo,zgeo,xgsm,ygsm,zgsm)
```

transforms_geo_to_gsm: GEO -> GSM system

```
input : xgeo,ygeo,zgeo   cartesian geo coordinates
output: xgsm,ygsm,zgsm   cartesian gsm coordinates
```

tgsmgeo

```
subroutine tgsmgeo(xgsm,ygsm,zgsm,xgeo,ygeo,zgeo)
```

transforms_gsm_to_geo*: GSM -> GEO system*

```
input : xgsm,ygsm,zgsm   cartesian gsm coordinates
output: xgeo,ygeo,zgeo   cartesian geo coordinates
```

tgeogse

```
subroutine tgeogse(xgeo,ygeo,zgeo,xgse,ygse,zgse)
```

transforms_geo_to_gse*: GEO -> GSE system*

```
input : xgeo,ygeo,zgeo   cartesian geo coordinates
output: xgse,ygse,zgse   cartesian gse coordinates
```

tgsegeo

```
subroutine tgsegeo(xgse,ygse,zgse,xgeo,ygeo,zgeo)
```

transforms_gse_to_geo*: GSE -> GEO system*

```
input : xgse,ygse,zgse   cartesian gse coordinates
output: xgeo,ygeo,zgeo   cartesian geo coordinates
```

tgeogsq

```
subroutine tgeogsq(xgeo,ygeo,zgeo,xgsq,ygsq,zgsq)
```

transforms_geo_to_gsq*: GEO -> GSEQ system*

```
input : xgeo,ygeo,zgeo   cartesian geo coordinates
output: xgsq,ygsq,zgsq   cartesian gsq coordinates
```

tgsqgeo

```
subroutine tgsqgeo(xgsq,ygsq,zgsq,xgeo,ygeo,zgeo)
```

transforms_gsq_to_geo: GSEQ-> GEO system

```
input : xgsq,ygsq,zgsq   cartesian gsq coordinates
output: xgeo,ygeo,zgeo   cartesian geo coordinates
```

tgeodme

```
subroutine tgeodme(xgeo,ygeo,zgeo,rlat,rlong,xdme,ydme,zdme)
```

transforms_geo_to_dme: *GEO -> DM system*

```
input : xgeo,ygeo,zgeo   cartesian geo coordinates
        rlat,rlong       latitude and longitude of the
                         point of observation (radians)

output: xdme,ydme,zdme   cartesian dme coordinates
```

tdmegeo

```
subroutine tdmegeo(xdme,ydme,zdme,rlat,rlong,xgeo,ygeo,zgeo)
```

transforms_dme_to_geo: *DM -> GEO system*

```
input : xdme,ydme,zdme   cartesian dme coordinates
        rlat,rlong       latitude and longitude of the
                         point of observation (radians)

output: xgeo,ygeo,zgeo   cartesian geo coordinates
```

tgeovdh

```
subroutine tgeovdh(xgeo,ygeo,zgeo,rlat,rlong,xvdh,yvdh,zvdh)
```

transforms_geo_to_vdh: *GEO -> VDH system*

```
input : xgeo,ygeo,zgeo   cartesian geo coordinates
        rlat,rlong       latitude and longitude of the
                         point of observation (radians)

output: xvdh,yvdh,zvdh   cartesian vdh coordinates
```

tvdhgeo

```
subroutine tvdhgeo(xvdh,yvdh,zvdh,rlat,rlong,xgeo,ygeo,zgeo)
```

transforms_vdh_to_geo: *VDH -> GEO system*

```
input : xvdh,yvdh,zvdh   cartesian vdh coordinates
        rlat,rlong       latitude and longitude of the
                         point of observation (radians)

output: xgeo,ygeo,zgeo   cartesian geo coordinates
```

tcarsph

```
subroutine tcarsph(x,y,z,r,teta,phi)
```

transforms_car_to_sph*: CAR -> SPH system*

```
input :   x,y,z         cartesian coordinates
output:   r,teta,phi    spherical coordinates  (radians)
```

tsphcar

```
subroutine tsphcar(r,teta,phi,x,y,z)
```

transforms_sph_to_car*: SPH -> CAR system*

```
input :   r,teta,phi    spherical coordinates  (radians)
output:   x,y,z         cartesian coordinates
```

6) description of "print information" subroutine

```
subroutine prinfo
```

print_informations *on the library; use gvernu*
result of this call is listed in section VI

```
input :   none
output:   print informations on terminal
```

VI - DIRECTION FOR USE

1) package delivered for installation

This section correspond to the file readme.txt of the delivered UNIX standard package; for DOS version, fortran source are named as *.for, and binary as *.obj; rocotexp, rocotinf and rocotche programs are detailed in the following paragraph.

```
ROCOTLIB Package for UNIX contents following files:

readme.txt      this file
rocotlib.o      binary       of  rocot library

rocotexp.f      fortran code of "example"     program
rocotexp.o      binary       of "example"     program
rocotexp.exe    executable   of "example"     program

rocotinf.f      fortran code of "information" program
rocotinf.o      binary       of "information" program
rocotinf.exe    executable   of "information" program
rocotinf.out    output       of "information" program

rocotche.f      fortran code of "check"       program
rocotche.o      binary       of "check"       program
rocotche.exe    executable   of "check"       program
rocotche.in     input        of "check"       program
rocotche.out    output       of "check"       program


rocotexp.exe can be used to see how use the library.

rocotinf.out contains any informations on the library;
it can be created again by:

rocotinf.exe >rocotinf.out

                       warning: original rocotinf.out is erased

rocotche program is a check program of the library, wich create
the output file rocotche.out from the input data in rocotche.in;
rocotche.out is created by:

rocotche.exe <rocotche.in

                       warning: original rocotche.out is erased

of course, it is possible to change input values into rocotche.in file
or directly answer by typing data on terminal.
obviously, rocotche.out is changed.

for new application, compilation and link can be as following:

f77 -c toto.f                        -->create toto.o
f77 toto.o rocotlib.o -o toto.exe  -->create toto.exe

Library has been developped on a UNIX machine with f77 compiler;
files rocotche.out correspond to results with UNIX FPS 350X machine;
a few difference can be observed on accuracy of results with another
machine.
                                                  P.Robert, november 92
```

2) example of Fortran user program

```
      program examp

      call redate(imonth,iday,iyear)
      call retime(ih,im,is)

      call codoty(imonth,iday,iyear,idoty)
      call ctimpa(iyear,idoty,ih,im,is)
      call gsundi(sxgei,sygei,szgei,sxgeo,sygeo,szgeo)
      call tgeigsm(sxgei,sygei,szgei,sxgsm,sygsm,szgsm)

      print*
      print*, 'Sun in GEI:',sxgei,sygei,szgei
      print*, 'Sun in GEO:',sxgeo,sygeo,szgeo
      print*, 'Sun in GSM:',sxgsm,sygsm,szgsm

      stop
      end
```

Fortran code file given above, and named rocotexp.f in standard package, can be compiled with a UNIX machine as:

```
f77 -c rocotexp.f
```

which produce rocotexp.o binary file

```
f77 rocotexp.o rocotlib.o -o rocotexp.exe
```

which produce rocotexp.exe executable file
(rocotexp.o can be then erased)

both command can be grouped as:

```
f77 rocotexp.f rocotlib.o -o rocotexp.exe
```

(with no rocotexp.o produced)

and program is running with:

```
rocotexp.exe
```

which give following results:

```
rpe3d-robert 4% rocotexp.exe
imonth,iday,iyear ? (ex: 10,17,1990)
7 14 1990
hour, minute, second ? (ex: 10,45,50)
12 0 0

Sun in GEI: -0.3712461      0.8519006      0.3693803
Sun in GEO:  0.9289786      2.3603529E-02  0.3693803
Sun in GSM:   1.000000     -1.4901161E-08        0.
STOP           statement executed
rpe3d-robert 5%
```

3) output of "print information" subroutine

The small fortran program below (rocotinf.f file in standard package) give the following output containing information, in particular the version number of the library.

```
      program rctinf
c
      call prinfo
c
      stop
      end
```

will give on default output (rocotinf.out in package):

```
Output produced by call prinfo:

*****************************************************

                EUROPEAN  SPACE  AGENCY

             Study of the Cluster Mission
               Planning Related Aspects
        within the Numerical Simulations Network
                ___________________

      Coordinates Transformation Library  ROCOTLIB

                    Version  1.1

          Patrick ROBERT - November 1992

*****************************************************

             AVAILABLE ROCOTLIB SUBROUTINES


    1) "Read and check" subroutines
        --------------------------

        example:
        call redate(imonth,iday,iyear)

    2) "Computation" subroutines
        -----------------------

        example:
        call codoty(imonth,iday,iyear,idoty)

    3) "Give" subroutines
        ----------------

        example:
        call gsundi(xgei,ygei,zgei,xgeo,ygeo,zgeo)

    4) "Transformations" subroutines
        ---------------------------

        example:
        call tgeigeo(xgei,ygei,zgei,xgeo,ygeo,zgeo)


WARNING: For use of 3) and 4) subroutines,
         the ctimpa(iyear,idoty,ih,im,is) subroutine
         must be called at each time  where the date
         and time are changed.
```

```
Example of program:

program examp

call redate(imonth,iday,iyear)
call retime(ih,im,is)

call codoty(imonth,iday,iyear,idoty)
call ctimpa(iyear,idoty,ih,im,is)
call gsundi(sxgei,sygei,szgei,sxgeo,sygeo,szgeo)
call tgeigsm(sxgei,sygei,szgei,sxgsm,sygsm,szgsm)

print*
print*, 'Sun in GEI:',sxgei,sygei,szgei
print*, 'Sun in GEO:',sxgeo,sygeo,szgeo
print*, 'Sun in GSM:',sxgsm,sygsm,szgsm

stop
end

For more information, see paper documentation

*****************************************************
```

4) check program of ROCOT library

general remarks

The rocotche.f and rocotche.exe programs delivered in the standard package allows the user to check the validity of the library. It run from an input date and time, an input vector test, and an input direction of observation. Next paragraph show examples of input and output data file (rocotche.in and rocotche.out files in standard package). Accuracy of the computation is one deduce of the Sun direction computation, which is 0.006 degrees.

Input date and time allows to:

I) check calendar conversions of the library;

II) set, list and test basic time parameters as Sun direction, Dipole direction, Ecliptic pole, Sun equator pole;

III) check basic transformation, for instance verify that direction of the Sun in GSM system is well the X axis, that Dipole direction in MAG system is well the Z axis, and so on;

Input vector test allows to:

IV) check the transformations given in the library, by changing the input vector across all available paths showed on the schematic diagram of § II-2.

Mainly, the vector is transformed:

a) as a "star" centred on GEI system (see schematic diagram);

b) as a "counter-clockwise ring cumulative transformation", from GEO to GEO after a complete circle;

c) as a "clockwise ring cumulative transformation", from GEO to GEO, in the other sense;

d) as a "star" centred on GEO system;

e) Finally, input direction of the point of observation is used to check local coordinate systems, i.e. DM and VDH.

At each "go and back", one check that the original vector is returned to the same value, taking into account accuracy of the computation (32 bits simple precision, 7 significative digits, and input Sun direction within 0.006 degrees).

input data file of check program

(file rocotche.in in the standard package)

```
10 17 1990
12 30 1
10. 30. 60.
45. 30.

----------------------------------------------------------------------
this file is a file example of input data for rocotche program

input:

imonth, iday, iyear
ihour,  imin, isec
R, teta, phi of anyone test vector (teta, phi in degrees)
geographic lat. and long. of point of observation for local systems test

----------------------------------------------------------------------
```

output result of check program

(file rocotche.out in the standard package)

```
              T E S T   O F   R O C O T L I B    -   Version  1.1

I) TEST OF ROCOTLIB CALENDAR CONVERSIONS
   -------------------------------------

INPUT DATE AND TIME:

month, day, year:             10    17  1990
hour, minute, second:         12    30     1

computed values:

decimal hour:                 12.50028
day of the year:             290
Julian day from 1-1-1950:  14900
leap year (1=yes,0=no):        0

recompute month and day from year and day of the year:    10    17
recompute date from Julian day  :    10    17  1990
recompute time from decimal hour:    12    30     1
```

suite --->

```
II) CHECK BASIC TIME PARAMETERS
    ---------------------------


SUN PARAMETERS IN GEI SYSTEM

Greenwich Sideral Time (deg.):   213.25262
ecliptic longitude     (deg.):   203.87914
right ascention        (deg.):   202.10098
declination            (deg.):    -9.26467

DIPOLE TILT ANGLE      (deg.):    -3.73804

SUN DIRECTION
                     in GEI:
                                X= -0.9144359E+00     r   =     0.1000000E+01
                                Y= -0.3713321E+00     teta=    99.26467 (deg.)
                                Z= -0.1609952E+00     phi =  -157.89903 (deg.)
                     in GEO:
                                X=  0.9683203E+00     r   =     0.1000000E+01
                                Y= -0.1908833E+00     teta=    99.26467 (deg.)
                                Z= -0.1609952E+00     phi =   -11.15164 (deg.)

DIPOLE DIRECTION
                     in GEI:
                                X= -0.1484070E+00     r   =     0.1000000E+01
                                Y=  0.1148030E+00     teta=    10.81440 (deg.)
                                Z=  0.9822402E+00     phi =   142.27560 (deg.)
                     in GEO:
                                X=  0.6115692E-01     r   =     0.1000000E+01
                                Y= -0.1773815E+00     teta=    10.81440 (deg.)
                                Z=  0.9822402E+00     phi =   -70.97702 (deg.)

ECLIPTIC DIRECTION
                     in GEI:
                                X=  0.0000000E+00     r   =     0.1000000E+01
                                Y= -0.3979853E+00     teta=    23.45229 (deg.)
                                Z=  0.9173918E+00     phi =   -90.00000 (deg.)
                     in GEO:
                                X=  0.2182279E+00     r   =     0.1000000E+01
                                Y=  0.3328196E+00     teta=    23.45229 (deg.)
                                Z=  0.9173918E+00     phi =    56.74738 (deg.)

SUN EQUATOR DIRECTION
                     in GEI:
                                X=  0.1218259E+00     r   =     0.1000000E+01
                                Y= -0.4233948E+00     teta=    26.14045 (deg.)
                                Z=  0.8977168E+00     phi =   -73.94758 (deg.)
                     in GEO:
                                X=  0.1302824E+00     r   =     0.1000000E+01
                                Y=  0.4208696E+00     teta=    26.14045 (deg.)
                                Z=  0.8977168E+00     phi =    72.79982 (deg.)
```

suite --->

```
III) CHECK BASIC TRANSFORMATIONS
     ---------------------------


     Check (S,E) angle is equal to 90 deg. : angle=    89.9949    (deg.)


1) input vector: Sun direction in GEI system
   -----------------------------------------

                                X= -0.9144359E+00     r   =     0.1000000E+01
                                Y= -0.3713321E+00     teta=    99.26467 (deg.)
                                Z= -0.1609952E+00     phi =  -157.89903 (deg.)

                               in GEO system

                                X=  0.9683203E+00     r   =     0.1000000E+01
                                Y= -0.1908833E+00     teta=    99.26467 (deg.)
                                Z= -0.1609952E+00     phi =   -11.15164 (deg.)


   check Y=0 in SM  system with tgeisma:

                                X=  0.9978815E+00     r   =     0.1000000E+01
                                Y=  0.2421439E-07     teta=    93.73015 (deg.)
                                Z= -0.6505735E-01     phi =     0.00000 (deg.)

                           with tgeosma:

                                X=  0.9978816E+00     r   =     0.1000000E+01
                                Y=  0.9313226E-08     teta=    93.73015 (deg.)
                                Z= -0.6505734E-01     phi =     0.00000 (deg.)

   check Y=0 and Z=0 in GSM system with tgeigsm:

                                X=  0.1000000E+01     r   =     0.1000000E+01
                                Y=  0.2421439E-07     teta=    90.00000 (deg.)
                                Z=  0.0000000E+00     phi =     0.00000 (deg.)

                                   with tgeogsm:

                                X=  0.1000000E+01     r   =     0.1000000E+01
                                Y=  0.9313226E-08     teta=    90.00000 (deg.)
                                Z=  0.1490116E-07     phi =     0.00000 (deg.)

   check Y=0 and Z=0 in GSE system with tgeigse:

                                X=  0.1000000E+01     r   =     0.1000000E+01
                                Y= -0.7450581E-08     teta=    89.99490 (deg.)
                                Z=  0.8901954E-04     phi =     0.00000 (deg.)

                                   with tgeogse:

                                X=  0.1000000E+01     r   =     0.1000000E+01
                                Y=  0.0000000E+00     teta=    89.99490 (deg.)
                                Z=  0.8901954E-04     phi =     0.00000 (deg.)

   check Y=0 and Z=0 in GSQ system with tgeigsq:

                                X=  0.1000000E+01     r   =     0.1000000E+01
                                Y= -0.2980232E-07     teta=    90.00000 (deg.)
                                Z=  0.1490116E-07     phi =     0.00000 (deg.)

                                   with tgeogsq:

                                X=  0.1000000E+01     r   =     0.1000000E+01
                                Y= -0.7450581E-08     teta=    90.00000 (deg.)
                                Z=  0.0000000E+00     phi =     0.00000 (deg.)
```

suite --->

```
2) input vector: Dipole direction in GEI system
   --------------------------------------------

                             X= -0.1484070E+00     r   =     0.1000000E+01
                             Y=  0.1148030E+00     teta=    10.81440 (deg.)
                             Z=  0.9822402E+00     phi =   142.27560 (deg.)

                               in GEO system

                             X=  0.6115692E-01     r   =     0.1000000E+01
                             Y= -0.1773815E+00     teta=    10.81440 (deg.)
                             Z=  0.9822402E+00     phi =   -70.97702 (deg.)


   check X=0 and Y=0 in MAG system with tgeimag:

                             X= -0.1490116E-07     r   =     0.1000000E+01
                             Y=  0.0000000E+00     teta=     0.00000 (deg.)
                             Z=  0.1000000E+01     phi =   180.00001 (deg.)

                                with tgeomag:

                             X= -0.1490116E-07     r   =     0.1000000E+01
                             Y=  0.0000000E+00     teta=     0.00000 (deg.)
                             Z=  0.1000000E+01     phi =   180.00001 (deg.)

   check X=0 and Y=0 in SM  system with tgeisma:

                             X= -0.1490116E-07     r   =     0.1000000E+01
                             Y= -0.1490116E-07     teta=     0.00000 (deg.)
                             Z=  0.1000000E+01     phi =  -135.00000 (deg.)

                                with tgeosma:

                             X=  0.0000000E+00     r   =     0.1000000E+01
                             Y= -0.1490116E-07     teta=     0.00000 (deg.)
                             Z=  0.1000000E+01     phi =   -90.00000 (deg.)

   check Y=0 in GSM system with tgeigsm:

                             X= -0.6505735E-01     r   =     0.1000000E+01
                             Y= -0.1490116E-07     teta=     3.73015 (deg.)
                             Z=  0.9978816E+00     phi =  -179.99999 (deg.)

                        with tgeogsm:

                             X= -0.6505734E-01     r   =     0.1000000E+01
                             Y= -0.1490116E-07     teta=     3.73015 (deg.)
                             Z=  0.9978816E+00     phi =  -179.99999 (deg.)
```

suite --->

```
3) input vector: Ecliptic direction in GEI system
   --------------------------------------------

                             X=  0.0000000E+00     r   =     0.1000000E+01
                             Y= -0.3979853E+00     teta=    23.45229 (deg.)
                             Z=  0.9173918E+00     phi =   -90.00000 (deg.)

                                 in GEO system

                             X=  0.2182279E+00     r   =     0.1000000E+01
                             Y=  0.3328196E+00     teta=    23.45229 (deg.)
                             Z=  0.9173918E+00     phi =    56.74738 (deg.)


   check X=0 and Y=0 in GSE system with tgeigse:

                             X=  0.8901954E-04     r   =     0.1000000E+01
                             Y=  0.0000000E+00     teta=     0.00510 (deg.)
                             Z=  0.1000000E+01     phi =     0.00000 (deg.)

                                with tgeogse:

                             X=  0.8901954E-04     r   =     0.1000000E+01
                             Y=  0.0000000E+00     teta=     0.00510 (deg.)
                             Z=  0.1000000E+01     phi =     0.00000 (deg.)

   check X=0 in GSQ system with tgeigsq:

                             X=  0.8901954E-04     r   =     0.9999999E+00
                             Y= -0.7816470E-01     teta=     4.48308 (deg.)
                             Z=  0.9969404E+00     phi =   -89.93475 (deg.)

                        with tgeogsq:

                             X=  0.8901954E-04     r   =     0.1000000E+01
                             Y= -0.7816467E-01     teta=     4.48308 (deg.)
                             Z=  0.9969406E+00     phi =   -89.93475 (deg.)




4) input vector: Sun equator in GEI system
   ---------------------------------------

                             X=  0.1218259E+00     r   =     0.1000000E+01
                             Y= -0.4233948E+00     teta=    26.14045 (deg.)
                             Z=  0.8977168E+00     phi =   -73.94758 (deg.)

                          in GEO system

                             X=  0.1302824E+00     r   =     0.1000000E+01
                             Y=  0.4208696E+00     teta=    26.14045 (deg.)
                             Z=  0.8977168E+00     phi =    72.79982 (deg.)


   check and Y=0 in GSQ system with tgeigsq:

                             X= -0.9871002E-01     r   =     0.1000000E+01
                             Y= -0.2980232E-07     teta=     5.66489 (deg.)
                             Z=  0.9951164E+00     phi =  -179.99999 (deg.)

                            with tgeogsq:

                             X= -0.9871002E-01     r   =     0.1000000E+01
                             Y=  0.0000000E+00     teta=     5.66489 (deg.)
                             Z=  0.9951165E+00     phi =   180.00001 (deg.)
```

suite --->

```
5) input vector: North geographic in GEO system
   --------------------------------------------

                             X=  0.0000000E+00     r   =     0.1000000E+01
                             Y=  0.0000000E+00     teta=     0.00000 (deg.)
                             Z=  0.1000000E+01     phi =     0.00000 (deg.)


   check X=0 and Y=0 in GEI system with tgeogei:

                             X=  0.0000000E+00     r   =     0.1000000E+01
                             Y=  0.0000000E+00     teta=     0.00000 (deg.)
                             Z=  0.1000000E+01     phi =     0.00000 (deg.)

   check Y=0 in MAG system with tgeimag:

                             X= -0.1876283E+00     r   =     0.1000000E+01
                             Y=  0.0000000E+00     teta=    10.81440 (deg.)
                             Z=  0.9822402E+00     phi =   180.00001 (deg.)

                        with tgeomag:

                             X= -0.1876283E+00     r   =     0.1000000E+01
                             Y=  0.0000000E+00     teta=    10.81440 (deg.)
                             Z=  0.9822402E+00     phi =   180.00001 (deg.)



IV) TEST OF ROCOTLIB TRANSFORMATION SUBROUTINES
    -------------------------------------------

1) input choosed test vector in GEO system :

                             X=  0.2500000E+01     r   =     0.1000000E+02
                             Y=  0.4330127E+01     teta=    30.00000 (deg.)
                             Z=  0.8660254E+01     phi =    60.00000 (deg.)

2) converts GEO system to GEI system by tgeogei

                             X=  0.2836924E+00     r   =     0.1000000E+02
                             Y= -0.4991945E+01     teta=    30.00000 (deg.)
                             Z=  0.8660254E+01     phi =   -86.74738 (deg.)
```

suite --->

```
a) STAR TRANSFORMATIONS AROUND  GEI  SYSTEM
   ----------------------------------------

3) converts GEI system to MAG system by tgeimag

                                X= -0.4845459E+01     r   =     0.1000000E+02
                                Y=  0.3774863E+01     teta=    37.89612 (deg.)
                                Z=  0.7891257E+01     phi =   142.07957 (deg.)

   come back to GEI system by tmaggei

                                X=  0.2836928E+00     r   =     0.1000000E+02
                                Y= -0.4991945E+01     teta=    30.00000 (deg.)
                                Z=  0.8660254E+01     phi =   -86.74738 (deg.)

   *** difference between first GEI vector: angle=     0.0000    (deg.)
                                            ratio=     0.9999999E+00


4) converts GEI system to SM  system by tgeisma

                                X=  0.7148904E+00     r   =     0.9999999E+01
                                Y=  0.6100573E+01     teta=    37.89612 (deg.)
                                Z=  0.7891257E+01     phi =    83.31633 (deg.)

   come back to GEI system by tsmagei

                                X=  0.2836916E+00     r   =     0.1000000E+02
                                Y= -0.4991944E+01     teta=    29.99999 (deg.)
                                Z=  0.8660254E+01     phi =   -86.74739 (deg.)

   *** difference between first GEI vector: angle=     0.0000    (deg.)
                                            ratio=     0.9999999E+00


5) converts GEI system to GSM system by tgeigsm

                                X=  0.1999916E+00     r   =     0.9999999E+01
                                Y=  0.6100573E+01     teta=    37.61735 (deg.)
                                Z=  0.7921048E+01     phi =    88.12238 (deg.)

   come back to GEI system by tgsmgei

                                X=  0.2836920E+00     r   =     0.9999999E+01
                                Y= -0.4991944E+01     teta=    29.99999 (deg.)
                                Z=  0.8660254E+01     phi =   -86.74738 (deg.)

   *** difference between first GEI vector: angle=     0.0000    (deg.)
                                            ratio=     0.1000000E+01


6) converts GEI system to GSE system by tgeigse

                                X=  0.1999916E+00     r   =     0.1000002E+02
                                Y=  0.1150798E+01     teta=     6.70771 (deg.)
                                Z=  0.9931566E+01     phi =    80.14131 (deg.)

   come back to GEI system by tgsegei

                                X=  0.2828839E+00     r   =     0.1000003E+02
                                Y= -0.4992280E+01     teta=    30.00179 (deg.)
                                Z=  0.8660128E+01     phi =   -86.75684 (deg.)

   *** difference between first GEI vector: angle=     0.0051    (deg.)
                                            ratio=     0.9999965E+00
```

suite --->

```
b) COUNTER-CLOCKWISE RING CUMULATIVE TRANSFORMATIONS
   ------------------------------------------------

   input choosed test vector in GEO system :

                                X=  0.2500000E+01     r   =     0.1000000E+02
                                Y=  0.4330127E+01     teta=    30.00000 (deg.)
                                Z=  0.8660254E+01     phi =    60.00000 (deg.)

2) converts GEO system to GEI system by tgeogei

                                X=  0.2836924E+00     r   =     0.1000000E+02
                                Y= -0.4991945E+01     teta=    30.00000 (deg.)
                                Z=  0.8660254E+01     phi =   -86.74738 (deg.)

7) converts GEI system to GSQ system by tgeigsq

                                X=  0.1999916E+00     r   =     0.1000000E+02
                                Y=  0.3709805E+00     teta=     2.41547 (deg.)
                                Z=  0.9991115E+01     phi =    61.67138 (deg.)

8) converts GSQ system to GSE system by tgsqgse

                                X=  0.1999916E+00     r   =     0.1000000E+02
                                Y=  0.1150798E+01     teta=     6.70773 (deg.)
                                Z=  0.9931549E+01     phi =    80.14131 (deg.)

9) converts GSE system to GSM system by tgsegsm

                                X=  0.1999916E+00     r   =     0.9978775E+01
                                Y=  0.6087644E+01     teta=    37.61759 (deg.)
                                Z=  0.7904211E+01     phi =    88.11839 (deg.)

10) converts GSM system to SM  system by tgsmsma

                                X=  0.7133721E+00     r   =     0.9965565E+01
                                Y=  0.6087644E+01     teta=    37.95540 (deg.)
                                Z=  0.7857746E+01     phi =    83.31636 (deg.)

11) converts SM  system to MAG system by tsmamag

                                X= -0.4835190E+01     r   =     0.9965564E+01
                                Y=  0.3766860E+01     teta=    37.95540 (deg.)
                                Z=  0.7857746E+01     phi =   142.07960 (deg.)

12) converts MAG system to GEO system by tmaggeo

                                X=  0.2493672E+01     r   =     0.9965565E+01
                                Y=  0.4323928E+01     teta=    30.05768 (deg.)
                                Z=  0.8625412E+01     phi =    60.02734 (deg.)

   *** difference between first GEO vector: angle=     0.0593    (deg.)
                                            ratio=     0.1003455E+01
```

suite --->

```
c) CLOCKWISE RING CUMULATIVE TRANSFORMATIONS
   -----------------------------------------

   input choosed test vector in GEO system :

                                X=  0.2500000E+01     r   =     0.1000000E+02
                                Y=  0.4330127E+01     teta=    30.00000 (deg.)
                                Z=  0.8660254E+01     phi =    60.00000 (deg.)

-12) converts GEO system to MAG system by tgeomag

                                X= -0.4845460E+01     r   =     0.1000000E+02
                                Y=  0.3774863E+01     teta=    37.89612 (deg.)
                                Z=  0.7891257E+01     phi =   142.07957 (deg.)

-11) converts MAG system to SM  system by tmagsma

                                X=  0.7148898E+00     r   =     0.1000000E+02
                                Y=  0.6100574E+01     teta=    37.89612 (deg.)
                                Z=  0.7891257E+01     phi =    83.31633 (deg.)

-10) converts SM  system to GSM system by tsmagsm

                                X=  0.1984798E+00     r   =     0.9986761E+01
                                Y=  0.6100574E+01     teta=    37.67555 (deg.)
                                Z=  0.7904366E+01     phi =    88.13656 (deg.)

-9) converts GSM system to GSE system by tgsmgse

                                X=  0.1984798E+00     r   =     0.9965565E+01
                                Y=  0.1156897E+01     teta=     6.76432 (deg.)
                                Z=  0.9896194E+01     phi =    80.26499 (deg.)

-8) converts GSE system to GSQ system by tgsegsq

                                X=  0.1984798E+00     r   =     0.9965564E+01
                                Y=  0.3798242E+00     teta=     2.46469 (deg.)
                                Z=  0.9956345E+01     phi =    62.41038 (deg.)

-7) converts GSQ system to GEI system by tgsqgei

                                X=  0.2875404E+00     r   =     0.9965563E+01
                                Y= -0.4982431E+01     teta=    30.05273 (deg.)
                                Z=  0.8625842E+01     phi =   -86.69708 (deg.)

   *** difference between first GEI vector: angle=     0.0584    (deg.)
                                            ratio=     0.1003456E+01


-2) converts GEI system to GEO system by tgeigeo

                                X=  0.2491565E+01     r   =     0.9965563E+01
                                Y=  0.4324281E+01     teta=    30.05273 (deg.)
                                Z=  0.8625842E+01     phi =    60.05031 (deg.)

   *** difference between first GEO vector: angle=     0.0584    (deg.)
                                            ratio=     0.1003456E+01
```

suite --->

```
d) STAR TRANSFORMATIONS AROUND  GEO  SYSTEM
   ----------------------------------------

   input choosed test vector in GEO system :

                                X=  0.2500000E+01     r   =     0.1000000E+02
                                Y=  0.4330127E+01     teta=    30.00000 (deg.)
                                Z=  0.8660254E+01     phi =    60.00000 (deg.)

13) converts GEO system to SM  system by tgeosma

                                X=  0.7148904E+00     r   =     0.9999999E+01
                                Y=  0.6100573E+01     teta=    37.89612 (deg.)
                                Z=  0.7891257E+01     phi =    83.31633 (deg.)

   come back to GEO system by tsmageo

                                X=  0.2500000E+01     r   =     0.1000000E+02
                                Y=  0.4330126E+01     teta=    29.99999 (deg.)
                                Z=  0.8660254E+01     phi =    60.00000 (deg.)

   *** difference between first GEO vector: angle=     0.0000    (deg.)
                                            ratio=     0.1000000E+01


14) converts GEO system to GSM system by tgeogsm

                                X=  0.1999917E+00     r   =     0.9999999E+01
                                Y=  0.6100573E+01     teta=    37.61735 (deg.)
                                Z=  0.7921048E+01     phi =    88.12238 (deg.)

   come back to GEO system by tgsmgeo

                                X=  0.2500000E+01     r   =     0.9999999E+01
                                Y=  0.4330126E+01     teta=    29.99999 (deg.)
                                Z=  0.8660254E+01     phi =    60.00000 (deg.)

   *** difference between first GEO vector: angle=     0.0000    (deg.)
                                            ratio=     0.1000000E+01


15) converts GEO system to GSE system by tgeogse

                                X=  0.1999917E+00     r   =     0.1000002E+02
                                Y=  0.1150798E+01     teta=     6.70771 (deg.)
                                Z=  0.9931566E+01     phi =    80.14130 (deg.)

   come back to GEO system by tgsegeo

                                X=  0.2500860E+01     r   =     0.1000003E+02
                                Y=  0.4329965E+01     teta=    30.00180 (deg.)
                                Z=  0.8660128E+01     phi =    59.99054 (deg.)

   *** difference between first GEO vector: angle=     0.0051    (deg.)
                                            ratio=     0.9999964E+00


16) converts GEO system to GSQ system by tgeogsq

                                X=  0.1999917E+00     r   =     0.1000000E+02
                                Y=  0.3709810E+00     teta=     2.41547 (deg.)
                                Z=  0.9991116E+01     phi =    61.67139 (deg.)

   come back to GEO system by tgsqgeo

                                X=  0.2500000E+01     r   =     0.1000000E+02
                                Y=  0.4330128E+01     teta=    30.00000 (deg.)
                                Z=  0.8660255E+01     phi =    60.00001 (deg.)

   *** difference between first GEO vector: angle=     0.0000    (deg.)
                                            ratio=     0.9999998E+00
```

```
e) LOCAL SYSTEMS
   -------------

   with direction of observation=input GEO vector
        geog. lat., long. =    60.00000    60.00000

17) converts GEO system to DM  system by tgeodme

                                X=  0.6142318E+01     r   =     0.1000000E+02
                                Y= -0.2384186E-06     teta=    37.89612 (deg.)
                                Z=  0.7891257E+01     phi =     0.00000 (deg.)

   come back to GEO system by tdmegeo

                                X=  0.2500000E+01     r   =     0.1000000E+02
                                Y=  0.4330128E+01     teta=    30.00001 (deg.)
                                Z=  0.8660254E+01     phi =    60.00000 (deg.)

   *** difference between first GEO vector: angle=     0.0000    (deg.)
                                             ratio=     0.9999999E+00

18) converts GEO system to VDH system by tgeovdh

                                X=  0.1000000E+02     r   =     0.1000000E+02
                                Y=  0.0000000E+00     teta=    90.00001 (deg.)
                                Z= -0.1430511E-05     phi =     0.00000 (deg.)

   come back to GEO system by tvdhgeo

                                X=  0.2500000E+01     r   =     0.1000000E+02
                                Y=  0.4330128E+01     teta=    30.00001 (deg.)
                                Z=  0.8660254E+01     phi =    60.00000 (deg.)

   *** difference between first GEO vector: angle=     0.0000    (deg.)
                                             ratio=     0.9999999E+00


   with anyone choosed direction of observation
        geog. lat., long. =    45.00000    30.00000

17-b) converts GEO system to DM  system by tgeodme

                                X=  0.5254447E+01     r   =     0.1000000E+02
                                Y=  0.3181014E+01     teta=    37.89612 (deg.)
                                Z=  0.7891257E+01     phi =    31.19049 (deg.)

   come back to GEO system by tdmegeo

                                X=  0.2500000E+01     r   =     0.1000000E+02
                                Y=  0.4330128E+01     teta=    30.00001 (deg.)
                                Z=  0.8660254E+01     phi =    60.00000 (deg.)

   *** difference between first GEO vector: angle=     0.0000    (deg.)
                                            ratio=     0.9999999E+00


18-b) converts GEO system to VDH system by tgeovdh

                                X=  0.9185587E+01     r   =     0.1000000E+02
                                Y=  0.2500000E+01     teta=    72.17046 (deg.)
                                Z=  0.3061862E+01     phi =    15.22516 (deg.)

   come back to GEO system by tvdhgeo

                                X=  0.2500000E+01     r   =     0.1000000E+02
                                Y=  0.4330128E+01     teta=    30.00000 (deg.)
                                Z=  0.8660254E+01     phi =    60.00001 (deg.)

   *** difference between first GEO vector: angle=     0.0000    (deg.)
                                            ratio=     0.9999999E+00

--------------------------------------------------------------------------------
end of test program
--------------------------------------------------------------------------------
```

5) Summary of available subroutine

"read and check" subroutines:

long name	call	arguments	comments
read_date	redate	(imonth,iday,iyear)	& check validity
read_time	retime	(ih,im,is)	& check validity

"computation" subroutines:

long name	call	arguments	comments
compute_time_parameters	ctimpa	(iyear,idoty,ihour,imin,isec)	prepare matrix
compute_dipole_direction	cdipdi	(iyear,idoty,d1,d2,d3)	in GEO system
compute_sun_direction	csundi	(iyear,idoty,ih,im,is,gst,slon,sra,sdec)	in GEI system
compute_leap_year	cleapy	(iyear,ileap)	ileap=1:leap year
compute_day_of_the_year	codoty	(imonth,iday,iyear,idoty)	idoty=1:January 1
compute_Julian_day	cjud50	(imonth,iday,iyear,jud50)	jud50=1:1,1,1950
compute_date_from_day_of_the_year	cdfdot	(iyear,idoty,imonth,iday)	and a given year
compute_date_from_Julian_day	cdfjud	(jud50,imonth,iday,iyear)	jud50=1:1,1,1950
compute_decimal_hour	codech	(ih,im,is,dech)	from ih,im,is
compute_time	cotime	(dech,ih,im,is)	from decimal hour
compute_angle_and_ratio	canara	(ux,uy,uz,vx,vy,vz,angle,ratio)	between U and V

"give" subroutines:

long name	call	arguments	comments
give_sun_direction	gsundi	(sxgei,sygei,szgei,sxgeo,sygeo,szgeo)	in GEI and GEO
give_dipole_direction	gdipdi	(dxgei,dygei,dzgei,dxgeo,dygeo,dzgeo)	in GEI and GEO
give_ecliptic_direction	gecldi	(exgei,eygei,ezgei,exgeo,eygeo,ezgeo)	in GEI and GEO
give_sun_rotation_direction	gsrodi	(rxgei,rygei,rzgei,rxgeo,rygeo,rzgeo)	in GEI and GEO
give_sun_parameter	gsunpa	(gmst,slon,sras,sdec)	in GEI (rad.)
give_dipole_tilt_angle	gdipta	(dipta)	in GSM (rad.)
give_version_number	gvernu	(vernu)	of the library

"transformation" subroutines:

long name	call	arguments	comments
transforms_gei_to_geo	tgeigeo	(xgei,ygei,zgei,xgeo,ygeo,zgeo)	GEI -> GEO
transforms_geo_to_gei	tgeogei	(xgeo,ygeo,zgeo,xgei,ygei,zgei)	GEO -> GEI
transforms_gei_to_mag	tgeimag	(xgei,ygei,zgei,xmag,ymag,zmag)	GEI -> MAG
transforms_mag_to_gei	tmaggei	(xmag,ymag,zmag,xgei,ygei,zgei)	MAG -> GEI
transforms_gei_to_sma	tgeisma	(xgei,ygei,zgei,xsma,ysma,zsma)	GEI -> SM
transforms_sma_to_gei	tsmagei	(xsma,ysma,zsma,xgei,ygei,zgei)	SM -> GEI
transforms_gei_to_gsm	tgeigsm	(xgei,ygei,zgei,xgsm,ygsm,zgsm)	GEI -> GSM
transforms_gsm_to_gei	tgsmgei	(xgsm,ygsm,zgsm,xgei,ygei,zgei)	GSM -> GEI
transforms_gei_to_gse	tgeigse	(xgei,ygei,zgei,xgse,ygse,zgse)	GEI -> GSE
transforms_gse_to_gei	tgsegei	(xgse,ygse,zgse,xgei,ygei,zgei)	GSE -> GEI
transforms_gei_to_gsq	tgeigsq	(xgei,ygei,zgei,xgsq,ygsq,zgsq)	GEI -> GSEQ
transforms_gsq_to_gei	tgsqgei	(xgsq,ygsq,zgsq,xgei,ygei,zgei)	GSEQ-> GEI
transforms_gse_to_gsq	tgsegsq	(xgse,ygse,zgse,xgsq,ygsq,zgsq)	GSE -> GSEQ
transforms_gsq_to_gse	tgsqgse	(xgsq,ygsq,zgsq,xgse,ygse,zgse)	GSEQ-> GSE
transforms_gse_to_gsm	tgsegsm	(xgse,ygse,zgse,xgsm,ygsm,zgsm)	GSE -> GSM
transforms_gsm_to_gse	tgsmgse	(xgsm,ygsm,zgsm,xgse,ygse,zgse)	GSM -> GSE
transforms_gsm_to_sma	tgsmsma	(xgsm,ygsm,zgsm,xsma,ysma,zsma)	GSM -> SM
transforms_sma_to_gsm	tsmagsm	(xsma,ysma,zsma,xgsm,ygsm,zgsm)	SM -> GSM
transforms_sma_to_mag	tsmamag	(xsma,ysma,zsma,xmag,ymag,zmag)	SM -> MAG
transforms_mag_to_sma	tmagsma	(xmag,ymag,zmag,xsma,ysma,zsma)	MAG -> SM
transforms_geo_to_mag	tgeomag	(xgeo,ygeo,zgeo,xmag,ymag,zmag)	GEO -> MAG
transforms_mag_to_geo	tmaggeo	(xmag,ymag,zmag,xgeo,ygeo,zgeo)	MAG -> GEO
transforms_geo_to_sma	tgeosma	(xgeo,ygeo,zgeo,xsma,ysma,zsma)	GEO -> SM
transforms_sma_to_geo	tsmageo	(xsma,ysma,zsma,xgeo,ygeo,zgeo)	SM -> GEO
transforms_geo_to_gsm	tgeogsm	(xgeo,ygeo,zgeo,xgsm,ygsm,zgsm)	GEO -> GSM
transforms_gsm_to_geo	tgsmgeo	(xgsm,ygsm,zgsm,xgeo,ygeo,zgeo)	GSM -> GEO
transforms_geo_to_gse	tgeogse	(xgeo,ygeo,zgeo,xgse,ygse,zgse)	GEO -> GSE
transforms_gse_to_geo	tgsegeo	(xgse,ygse,zgse,xgeo,ygeo,zgeo)	GSE -> GEO
transforms_geo_to_gsq	tgeogsq	(xgeo,ygeo,zgeo,xgsq,ygsq,zgsq)	GEO -> GSEQ
transforms_gsq_to_geo	tgsqgeo	(xgsq,ygsq,zgsq,xgeo,ygeo,zgeo)	GSEQ-> GEO
transforms_geo_to_dme	tgeodme	(xgeo,ygeo,zgeo,rlat,rlong,xdme,ydme,zdme)	GEO -> DM
transforms_dme_to_geo	tdmegeo	(xdme,ydme,zdme,rlat,rlong,xgeo,ygeo,zgeo)	DM -> GEO
transforms_geo_to_vdh	tgeovdh	(xgeo,ygeo,zgeo,rlat,rlong,xvdh,yvdh,zvdh)	GEO -> VDH
transforms_vdh_to_geo	tvdhgeo	(xvdh,yvdh,zvdh,rlat,rlong,xgeo,ygeo,zgeo)	VDH -> GEO
transforms_car_to_sph	tcarsph	(x,y,z,r,teta,phi)	CAR -> SPH
transforms_sph_to_car	tsphcar	(r,teta,phi,x,y,z)	SPH -> CAR

"print information" subroutine

long name	call	arguments	comments
print_informations	prinfo	none	

Bibliography

Geophysical coordinate transformations, C.T. Russell, cosmic electrodynamics, v.2, 184-196, 1971.

NOTES

NOTES